Advanced Nanotechnology

Chung-Ting Ke

02/19/2025

Classroom: P101, 1F, IoP

Class hours: Wednesday, 09:10-12:00

TiGP@AS

Course Syllabus

Date	Topic	Instructor
2/19	Introduction and review	Chung-Ting Ke
2/26	Charge transport review and nanoelectronics	Chung-Ting Ke
3/5	Carbon-based nanomaterials	Chung-Ting Ke
3/12	Superconductivity	Tien-Ming Chuang
3/19	Midterm exam I	Chung-Ting Ke
3/26	Quantum transport	Chen-Hsuan Hsu
4/2	Superconducting theory and devices	Chung-Ting Ke
4/9	Magnetism and magnetoelectronics (1)	Shang Fan Lee
4/16	Magnetism and magnetoelectronics (2)	Shang Fan Lee
4/23	2D materials	Chung-Ting Ke
4/30	Thermoelectricity in nanoscale systems	Ou Min-Nan
5/07	Nano-optics	Yu-Chieh Wen
5/14	Midterm II	Chung-Ting Ke
5/21	Topological materials 1	Tien-Ming Chuang
5/28	Quantum Computation	Chung-Ting Ke
6/4	Study group oral presentation (I)	Chung-Ting Ke
6/11	Study group oral presentation (II)	Chung-Ting Ke

Text book and Evaluation

Textbook:

Nanostructures and nanotechnology by Douglas Natelson, Cambridge University Press (2015)
eBook available (https://doi.org/10.1017/CBO9781139025485) if accessed from AS domain.

Evaluation:

Quiz and attendance 30%

Midterm I 20%

Midterm II 20%

Final presentation 40%

Let’s get to know each other

Chung-Ting Ke (Ting) 柯忠廷

Assistant Research Fellow in the Institute of Physics and RCCI

Research Field：
Quantum transport, Quantum materials
Topological system, Superconducting qubit

Expertise：
Low-temperature physics, low noise electrical measurement
Nanofabrication, 2D materials.

Charge transport and nanoelectronics I&II

TIGP Advanced Nanotechnology

Chung-Ting Ke

02/19/2025

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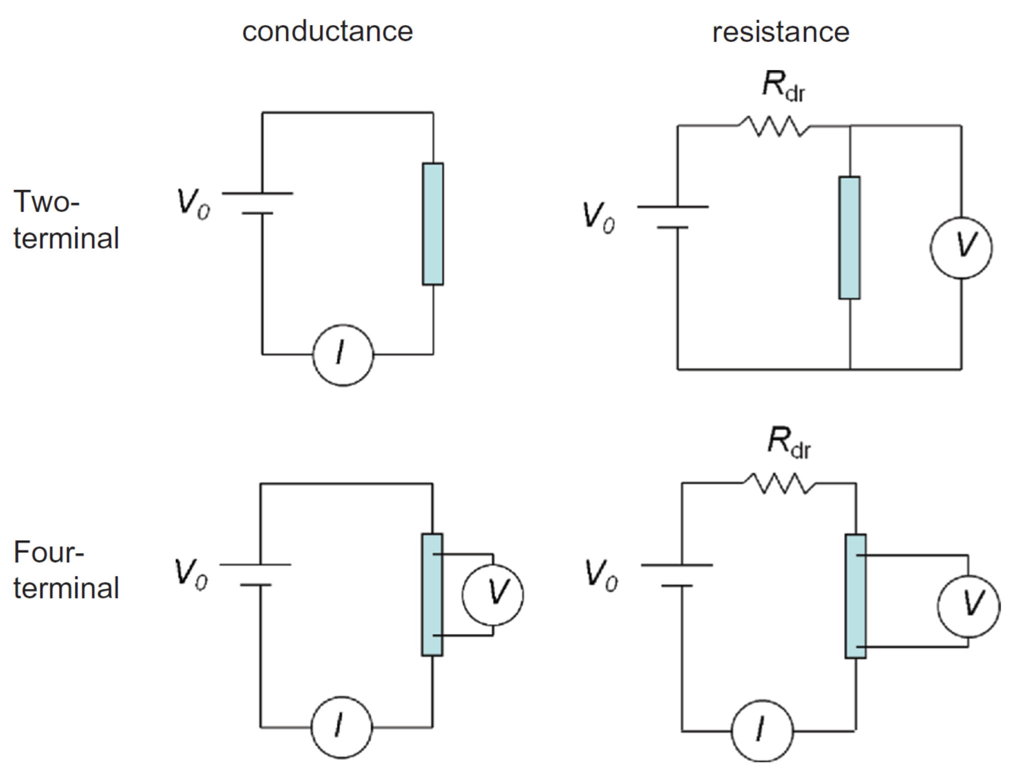
Charge transport and nanoelectronics

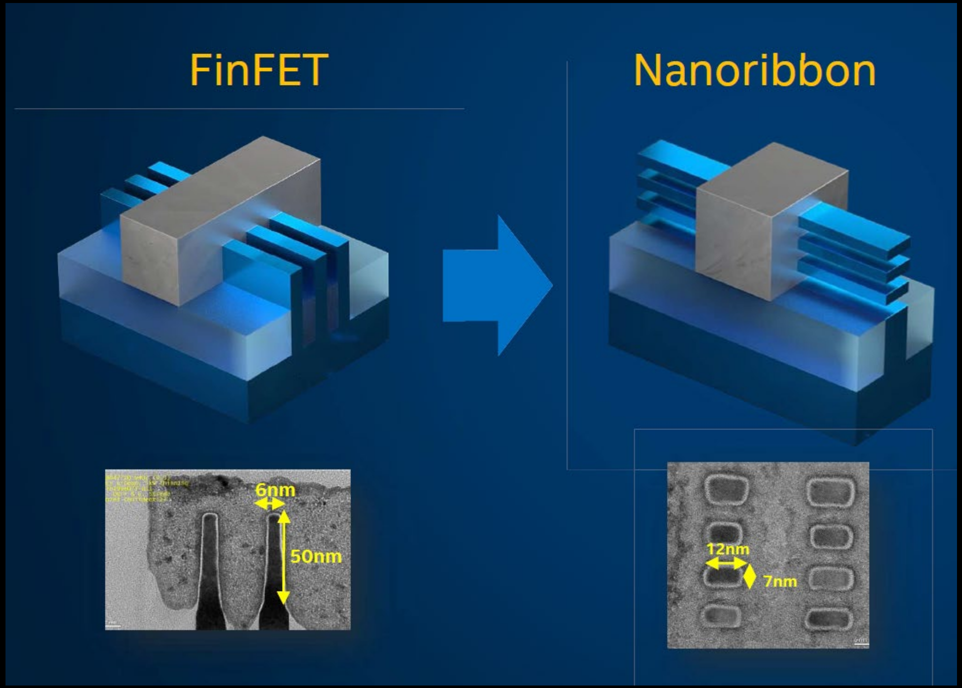
What is the charge transport

Charge moving—electrical response



Nanoelectronics

"Nano" means a scale — however,
reference length scale is important!



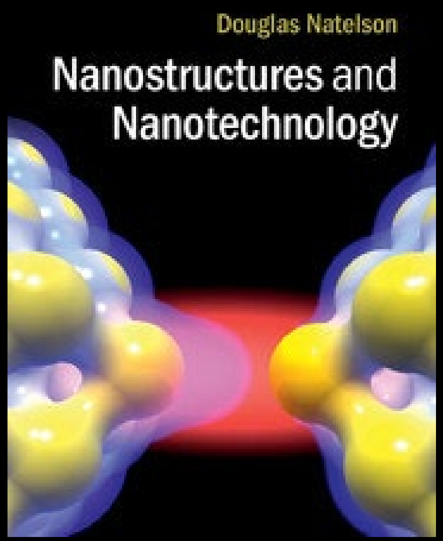
intel

Outline – chapter 2-4

2.1 Free electrons
2.2 Nearly free electrons
2.3 Chemical approaches to electronic structure
2.4 More modern electronic structure methods
2.5 Lattice dynamics: phonons

3.1 Electronic types of solids
3.2 Metals
3.3 Inorganic semiconductors
3.4 Band insulators
3.5 Correlated oxides
3.6 Molecular structures

4.1 Characterization
4.2 Materials growth
4.3 Material removal
4.4 Patterning



Outline – chapter 5-6

5.1 Defects

5.2 Interfaces and surfaces

5.3 Screening

5.4 Excitons

5.5 Junctions between materials

5.6 Quantum wires

5.7 Quantum dots

6.1 Transport terminology

6.2 Kinetic concepts

6.3 Hall effect

6.4 Quantum transport

6.5 The classical MOSFET

6.6 State-of-the-art

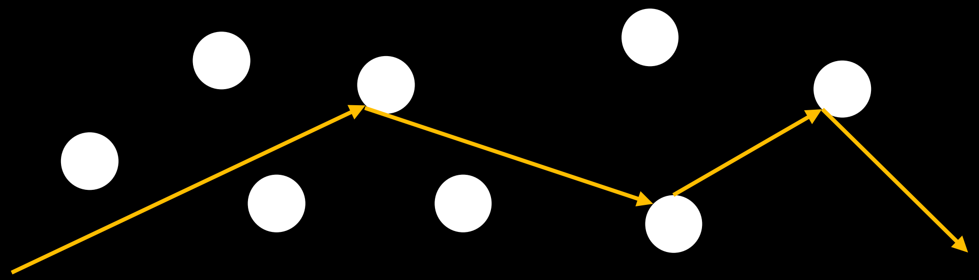
6.7 Beyond CMOS

Transport 101

Earlier studies on metals are trying to understand electrons' behavior in metal materials

- Drude model

scattering mechanism happening between ions and electrons



DC conductivity can be described as

$$\boldsymbol{j} = \sigma \boldsymbol{E}, \sigma = \frac{ne^2\tau}{m}$$

Where $\boldsymbol{j}$ is the current density, $\boldsymbol{\sigma}$ is the conductance, $\boldsymbol{E}$ is the electric field $\boldsymbol{n}$ is the charge density $\boldsymbol{m}$ is the electron mass and $\boldsymbol{\tau}$ is the average time constant

Ref: Ashcroft/Mermin ch1-3 and Douglas Natelson ch2

Transport 101

We would like to know the behavior of the electron(s)

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial x^2}\right)\psi(r)=\varepsilon\psi(r)$$

- Free electron (Drude-Sommerfeld theory)

  Simple, it can explain a lot of physics to a certain degree.
  Several assumptions:

  1. the interaction of ions and electrons away from the Fermi level is neglected.
  2. electron-electron interaction is neglected.
  3. The outcome of collision is treated with an average collision without considering the detail of the collision.
  4. the electron distribution follows the Fermi-Dirac distribution.

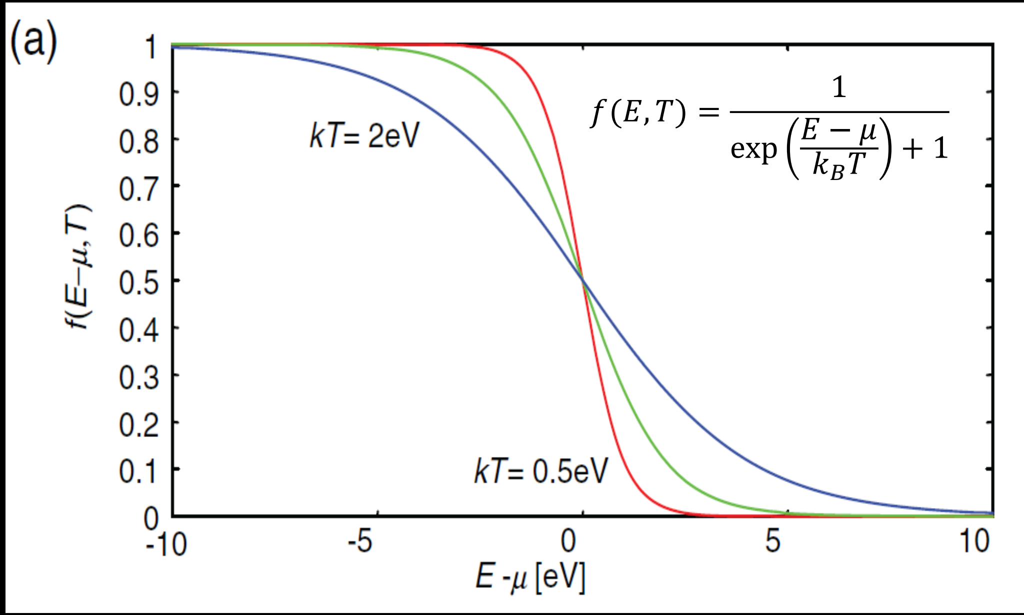
  Despite the oversimplification, the free electron model has a great success in understanding a lot of phenomena
  For example density of state, heat transport, Wiedemann-Franz law, etc.

Ref: Ashcroft/Mermin ch2/ch3 and Natelson ch2

Transport 101

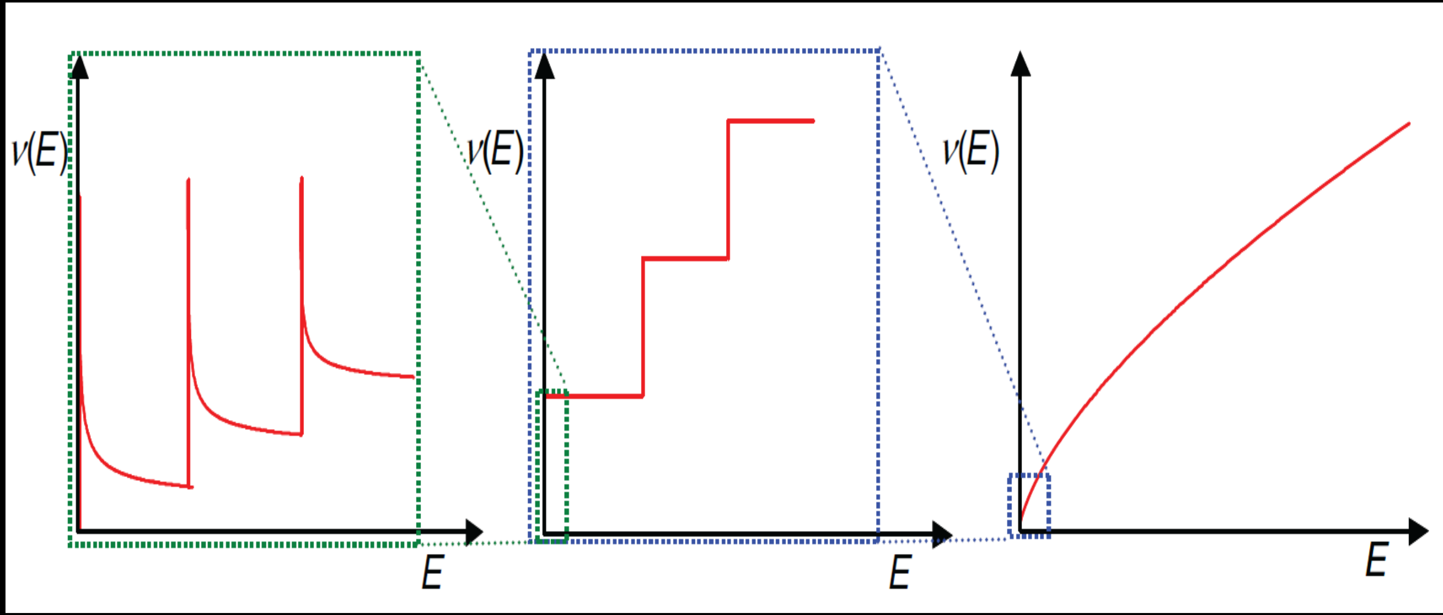
Fermi Dirac distribution

For electron gas– Maxwell distribution doesn't work, Pauli exclusion leads to a Fermi Dirac distribution



Density of state

For a small system, the allowed state to fill in a single particle is greatly depending on the confinement.



Ref: Ashcroft/Mermin ch2/ch3 and Natelson ch2

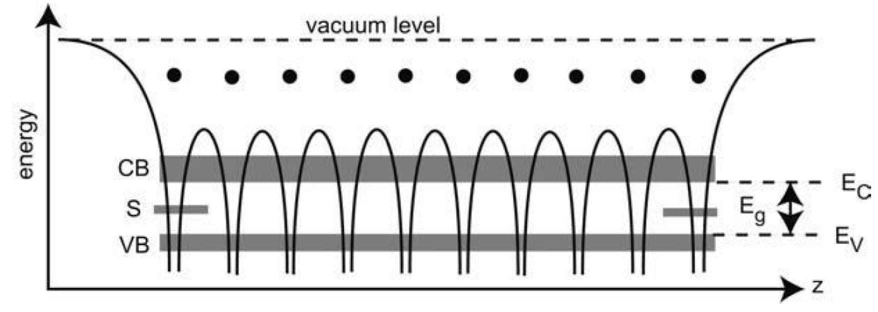
Transport 101

Degrees of freedom	Dispersion (kinetic energy)	Density of states	Effective density of states
3 (bulk)	$E = \frac{\hbar^2}{2m^*}(k_x^2 + k_y^2 + k_z^2)$	$\rho_{\text{DOS}}^{\text{3D}} = \frac{1}{2\pi^2}\left(\frac{2m^*}{\hbar^2}\right)^{\frac{3}{2}}\sqrt{E - E_{\text{C}}}$	$N_{\text{c}}^{\text{3D}} = \frac{1}{\sqrt{2}}\left(\frac{m^* kT}{\pi\hbar^2}\right)^{\frac{3}{2}}$
2 (slab)	$E = \frac{\hbar^2}{2m^*}(k_x^2 + k_y^2)$	$\rho_{\text{DOS}}^{\text{2D}} = \frac{m^*}{\pi\hbar^2}\sigma(E - E_{\text{C}})$	$N_{\text{c}}^{\text{2D}} = \frac{m^*}{\pi\hbar^2}kT$
1 (wire)	$E = \frac{\hbar^2}{2m^*}(k_x^2)$	$\rho_{\text{DOS}}^{\text{1D}} = \frac{m^*}{\pi\hbar}\sqrt{\frac{m^*}{2(E - E_{\text{C}})}}$	$N_{\text{c}}^{\text{1D}} = \sqrt{\frac{m^* kT}{2\pi\hbar^2}}$
0 (box)	–	$\rho_{\text{DOS}}^{\text{0D}} = 2\delta(E - E_{\text{C}})$	$N_{\text{c}}^{\text{0D}} = 2$

GT, ECE course

Transport 101

$U(r)$= $U(r+R)$



Bloch's Theorem $\left(-\frac{\hbar^2}{2m}\nabla^2 + U(r)\right)\psi(r) = \varepsilon\psi(r)$

$\boldsymbol{\varphi}(\boldsymbol{r}) = e^{ikr}u_k(r)$

In a periodic potential created by ions or valence electrons.
We define a translation function T_R.

$$\boldsymbol{T_R f}(\boldsymbol{r}) = f(r+R) \quad \boldsymbol{T_R \varphi}(\boldsymbol{r}) = \boldsymbol{c}(\boldsymbol{R})\boldsymbol{\varphi}(r) \text{ and } \quad \boldsymbol{c}(\boldsymbol{R}) = \boldsymbol{exp}(\boldsymbol{ik}\cdot\boldsymbol{R})$$

Now, we realize that $\boldsymbol{T_R \varphi}(\boldsymbol{r}) = \boldsymbol{\varphi}(r+R) = \boldsymbol{exp}(\boldsymbol{ik}\cdot\boldsymbol{R})\boldsymbol{\varphi}(r)$
And we can define $u_k(r) = \exp(-ikr)\boldsymbol{\varphi}(r)$
Working out the math, we can get $u_k(r+R) = \exp\big(-ik(r+R)\big)\,\boldsymbol{\varphi}(r+R)$

$$= \exp(-ikr)\,\boldsymbol{\varphi}(r) = u_k(r)$$

Ref: Ashcroft/Mermin ch2/ch3 and Natelson ch2, Heinzel ch2

Transport 101

Band Structure

with the simple single-particle wavefunction, we can further impose the boundary condition, a Born-von Karman B.Cs. assume periodicity happened in a_i direction after N_i site.

$$\boldsymbol{\varphi}(r+R), let\ R = a_i N_i,$$

R is also the Bravais lattice vector(recall of reciprocal lattice!)

If we use a cube lattice, we can have i=1-3, for three directions.

Bloch's theorem: $\boldsymbol{\varphi}(r+R) = \exp(ika_iN_i) = \exp(2\pi i x_i N_i) = 1$, note $kR = 2\pi j$

From this equality, we know that the allowed k values,

$$\mathrm{k} = \sum_{i=1}^{3} \frac{j_i}{N_i} b_i$$

Meaning, different to free electron case, multiple solutions!

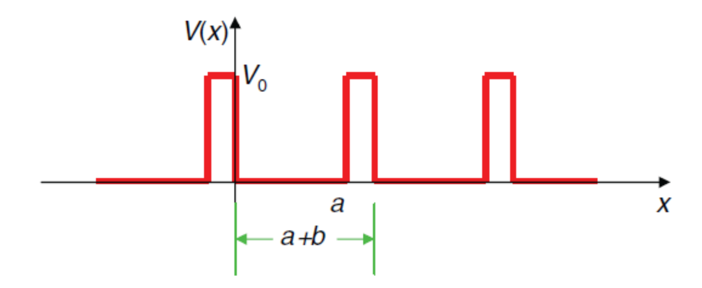
Recap:

The primitive lattice vector $\mathrm{R} = \sum_{i=1}^{3} n_i a_i$

The reciprocal lattice vector $\mathrm{G} = \sum_{i=1}^{3} n_i b_i$

Ref: Natelson ch2

Transport 101



1D Band Structure (König Penney model)

Now we can calculate the simple 1D band structure under a periodic potential,

$$\psi(x) = A\mathrm{e}^{\mathrm{i}\alpha x} + B\mathrm{e}^{-\mathrm{i}\alpha x},$$
$$\alpha \equiv \left[\frac{2mE}{\hbar^2}\right]^{1/2},$$
$$a > x > 0,$$

$$\psi(x) = C\mathrm{e}^{\beta x} + D\mathrm{e}^{-\beta x},$$
$$\beta \equiv \left[\frac{2m}{\hbar^2}(V_0 - E)\right]^{1/2},$$
$$-b < x < 0.$$

Bloch's theorem $\psi(x) = e^{ikr}u_k(x)$

$$u_k(x) = A\mathrm{e}^{\mathrm{i}(\alpha-k)x} + B\mathrm{e}^{-\mathrm{i}(\alpha+k)x}, \quad a > x > 0,$$
$$u_k(x) = C\mathrm{e}^{\mathrm{i}(\beta-\mathrm{i}k)x} + D\mathrm{e}^{-\mathrm{i}(\beta+\mathrm{i}k)x}, \quad -b < x < 0.$$

Working out the continuity conditions $u_k(x), du_k(x)/dx$ and periodic in a+b

One arrives at the energy-k relationship

$$\cos(\alpha a)\cosh(\beta b) + \frac{\beta^2 - \alpha^2}{2\alpha\beta}\sin(\alpha a)\sinh(\beta b) = \cos(k(a+b)).$$

For E> V_0 and V_0 goes to ∞ and b goes to 0,

$$\cos(ka) = \cos(\alpha a) + (p\sin(\alpha a))/(\alpha a) \qquad \tfrac{1}{2}\beta ab = p$$

https://lampx.tugraz.at/~hadley/ss1/KronigPenney/KronigPenney.php

Natelson Ch2

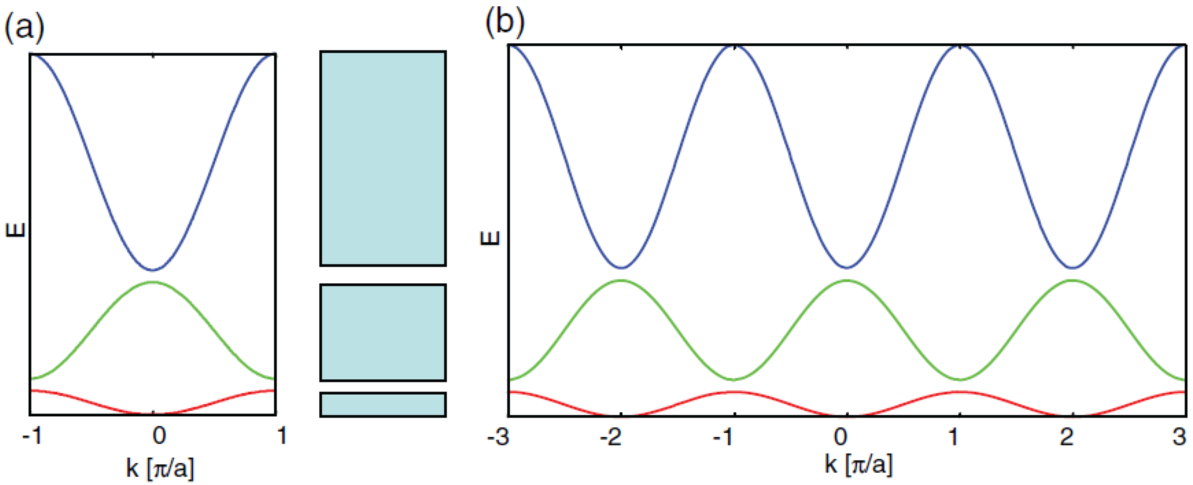
Transport 101

1D Band Structure (König Penney model)

From the previous calculation, energy as the function of k: $F(E) = \cos(k(a+b))$
For $|F(E)| > 1$, k will have to be imaginary, therefore, it is forbidden.
Under a weak potential case and more electrons, one can arrive the 1D band structure.
(See Ashcroft/Mermin ch8 to ch9)

First Brillouin zone

Repeating zone



Again, one can prove that outside the FBZ is only repeating :

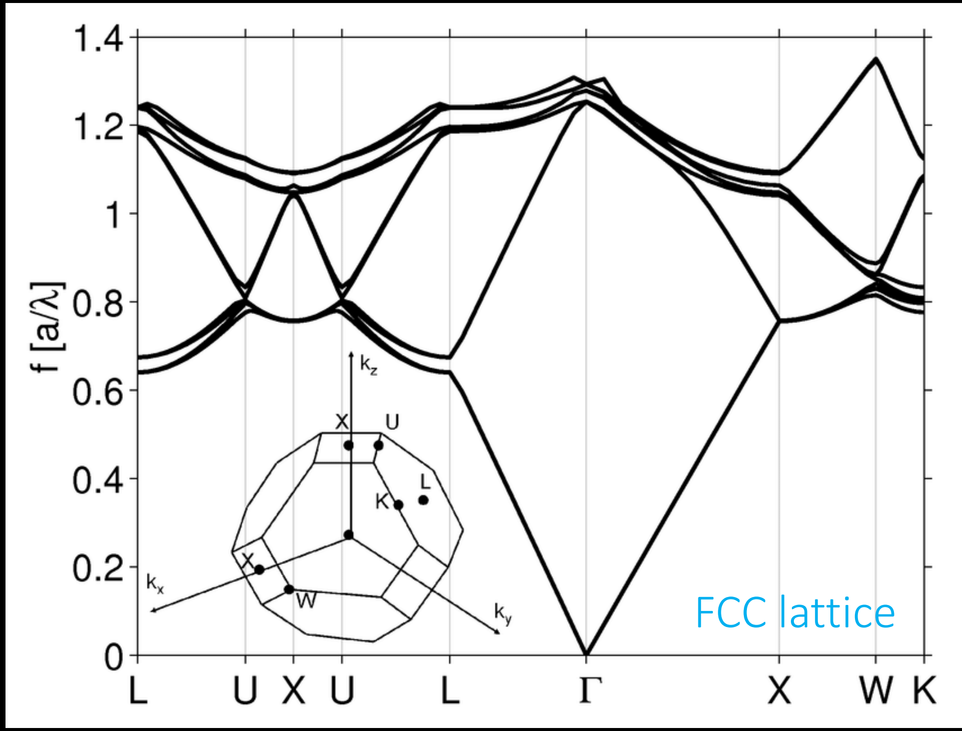
$$\begin{aligned}\psi_{n,\mathbf{k}+\mathbf{G}}(\mathbf{r}) &= \exp(\mathrm{i}(\mathbf{k}+\mathbf{G})\cdot\mathbf{r})u_{n,\mathbf{k}+\mathbf{G}}(\mathbf{r}) \\ &= \exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r})[u_{n,\mathbf{k}+\mathbf{G}}(\mathbf{r})\exp(\mathrm{i}\mathbf{G}\cdot\mathbf{r})] \\ &= \exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r})\tilde{u}(\mathbf{r}) \\ &= \psi_{n',\mathbf{k}}(\mathbf{r}).\end{aligned}$$

Transport 101

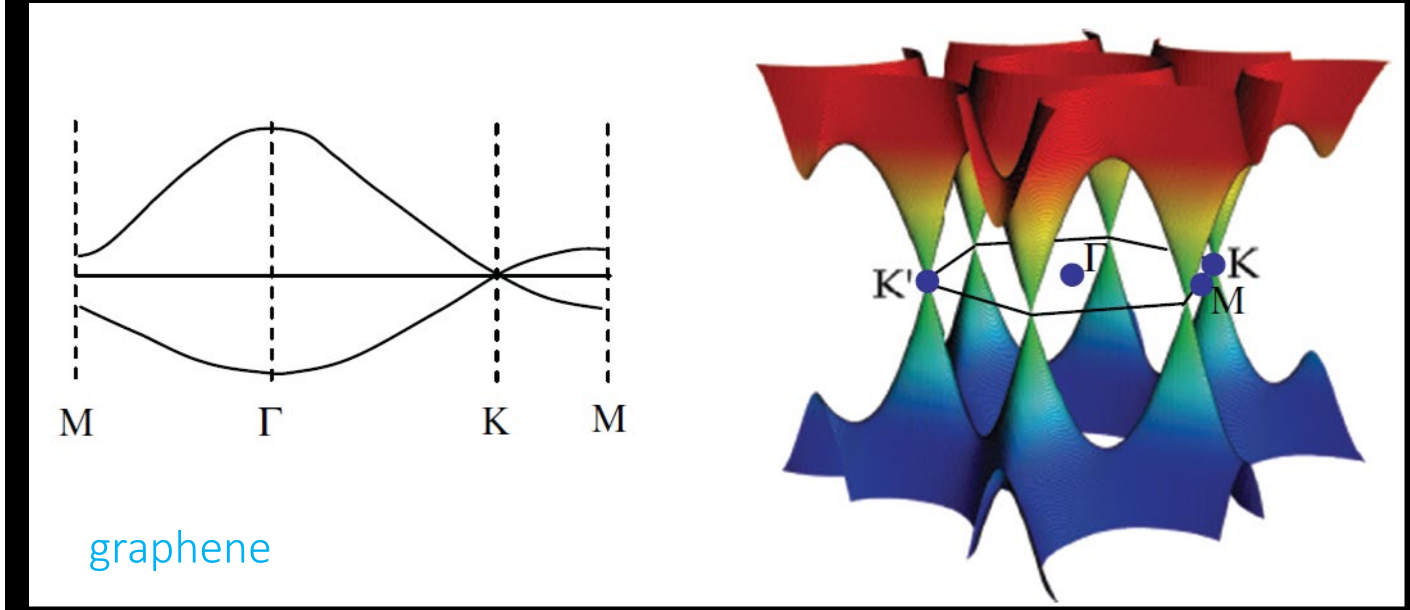
Other methods to calculate the band structure:

Tight-Binding model, k p theory(effective mass/spin-orbit), green function, etc.

Exact band structure is much more complicated!



Celine Vion et al. JAP (2009)



Ref: http://lampx.tugraz.at/~hadley/ss1/empty/empty.php

Transport 101

Density of levels and van hove singularity:

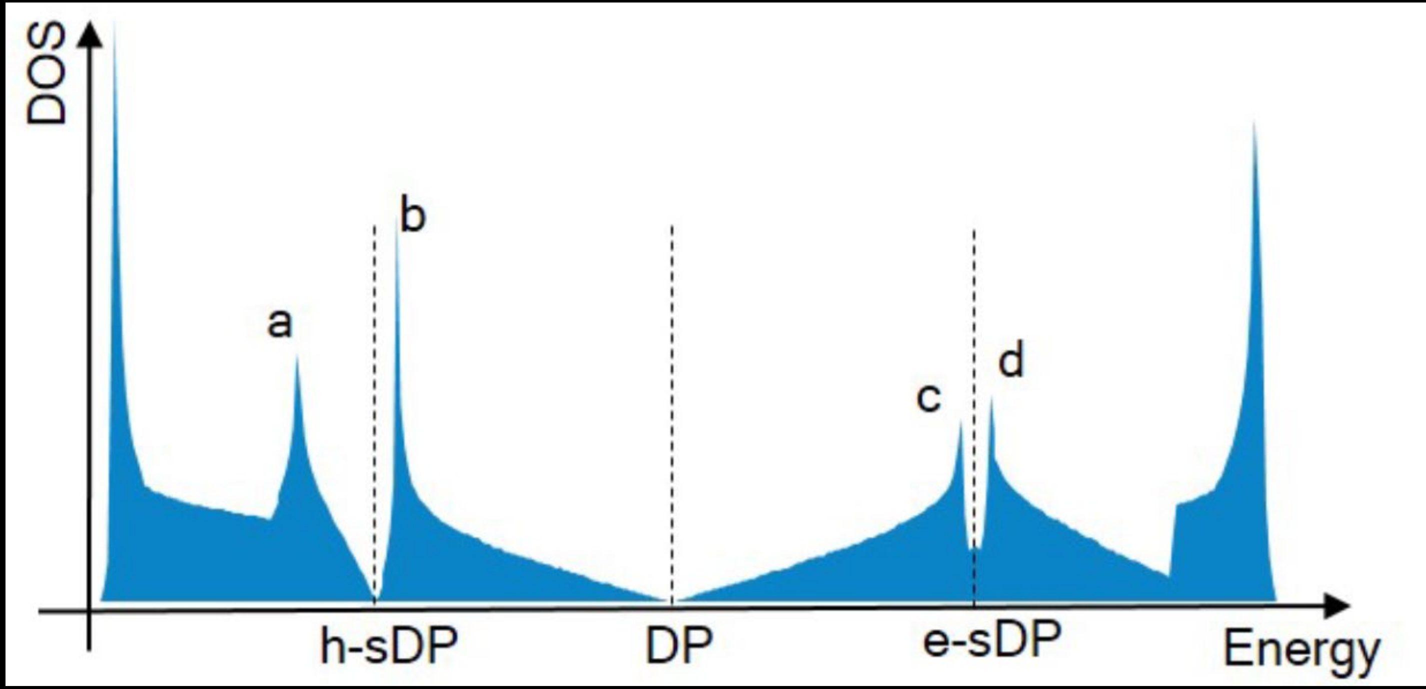
We can calculate the weighted sum over the electronic levels, like DOS in the free electron case. (A&M Ch8)

$$g_n(\varepsilon)d\varepsilon = \int_{S_n(\varepsilon)} \frac{dS}{4\pi^3}\delta k(\mathbf{k})$$

$$\delta k(\mathbf{k}) \ = \frac{d\varepsilon}{|\nabla\varepsilon|}$$

$$g_n(\varepsilon) = \int_{S_n(\varepsilon)} \frac{dS}{4\pi^3}\frac{1}{|\nabla\varepsilon|}$$

When $|\nabla\varepsilon|$=0 we have a divergence of $dg_n/d\varepsilon$ is called van Hove singularities



Lei Wang, et al, Science Advance 2016

Transport 101

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$$\delta k(\mathbf{k}) = \frac{d\varepsilon}{|\nabla\varepsilon|}$$

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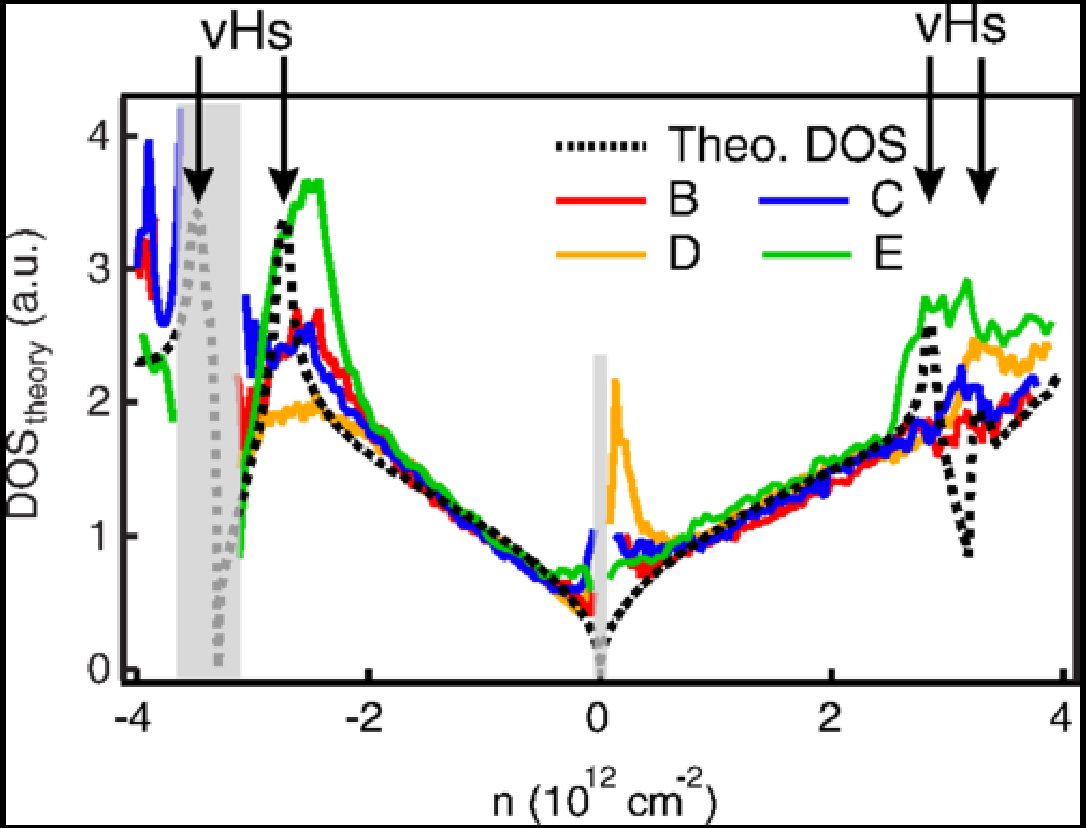


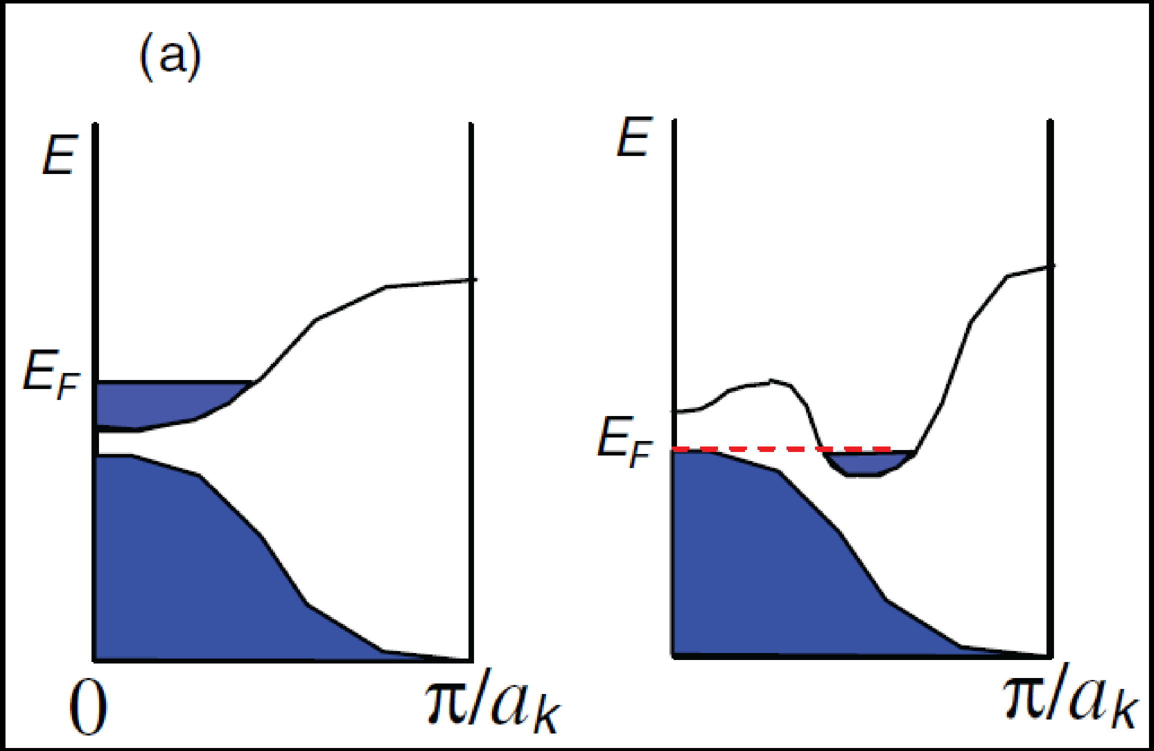
D.I. Indolese et al PRL 2018

Type of materials

Metal
Insulator
Semiconductor
Superconductor
Topological material

Type of materials

Metal
Insulator
Semiconductor
Superconductor
Topological material



Consider e/h as quasiparticle states
with a lifetime τ, so energy uncertainty, $\Gamma \sim \hbar/\tau$, to have a well-defined quasiparticles
$\Gamma \ll k_B T$. Under certain conditions, we can treat them as non-interacting Fermi liquid.

Type of materials

Metal
Insulator
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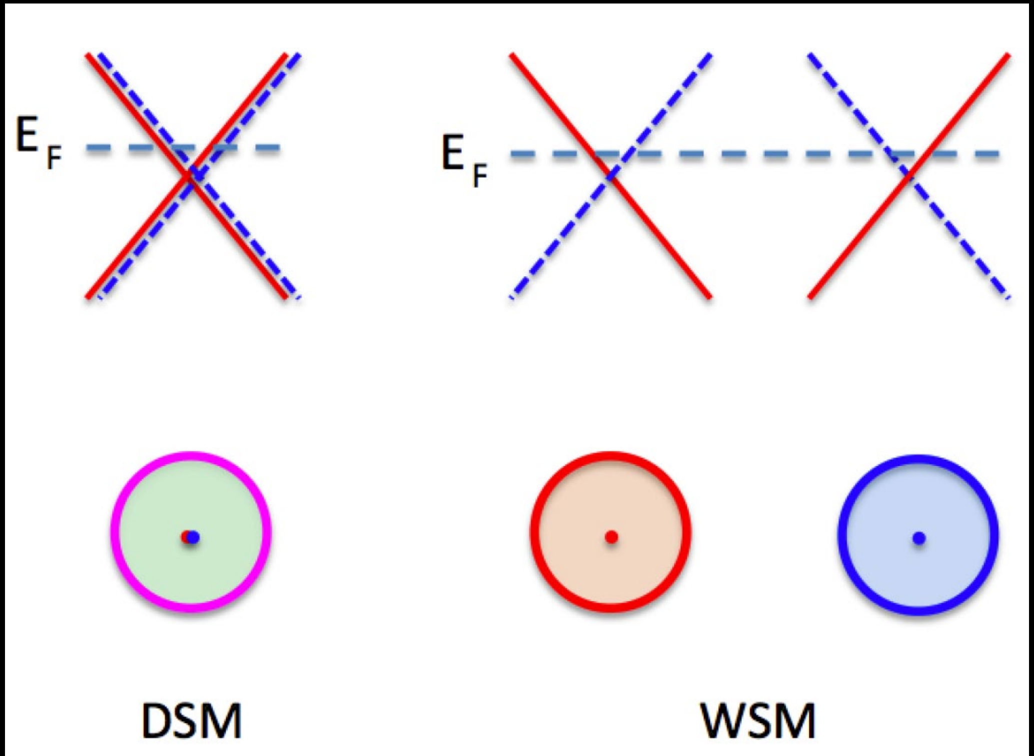
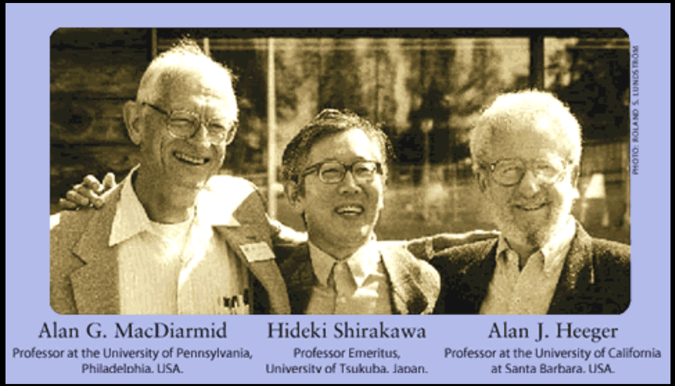
Typical metal:
Akali, Nobel, transition and
Rare earth metals.

Yet, there are also organic metals!!
2000 Nobel prize in chemistry

Semimetal





Type of materials

Metal
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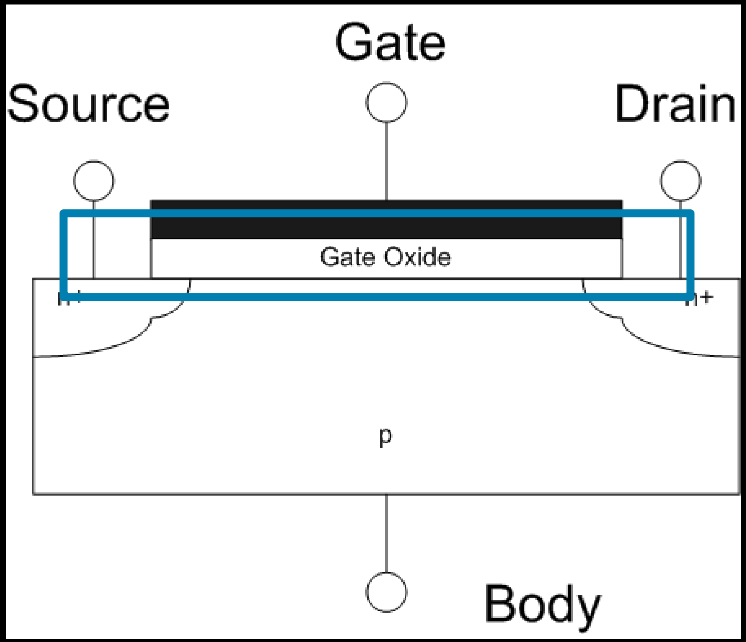
Semiconductor
Superconductor
Topological material

One of the key elements in many mesoscopic devices. For example, MOSFET can use high k materials for the gate dielectric, allowing a much better gate response and lower leakage.



Potential materials: HfO_4Si, HfO_2 etc.

Type of materials

Metal
Insulator
Semiconductor
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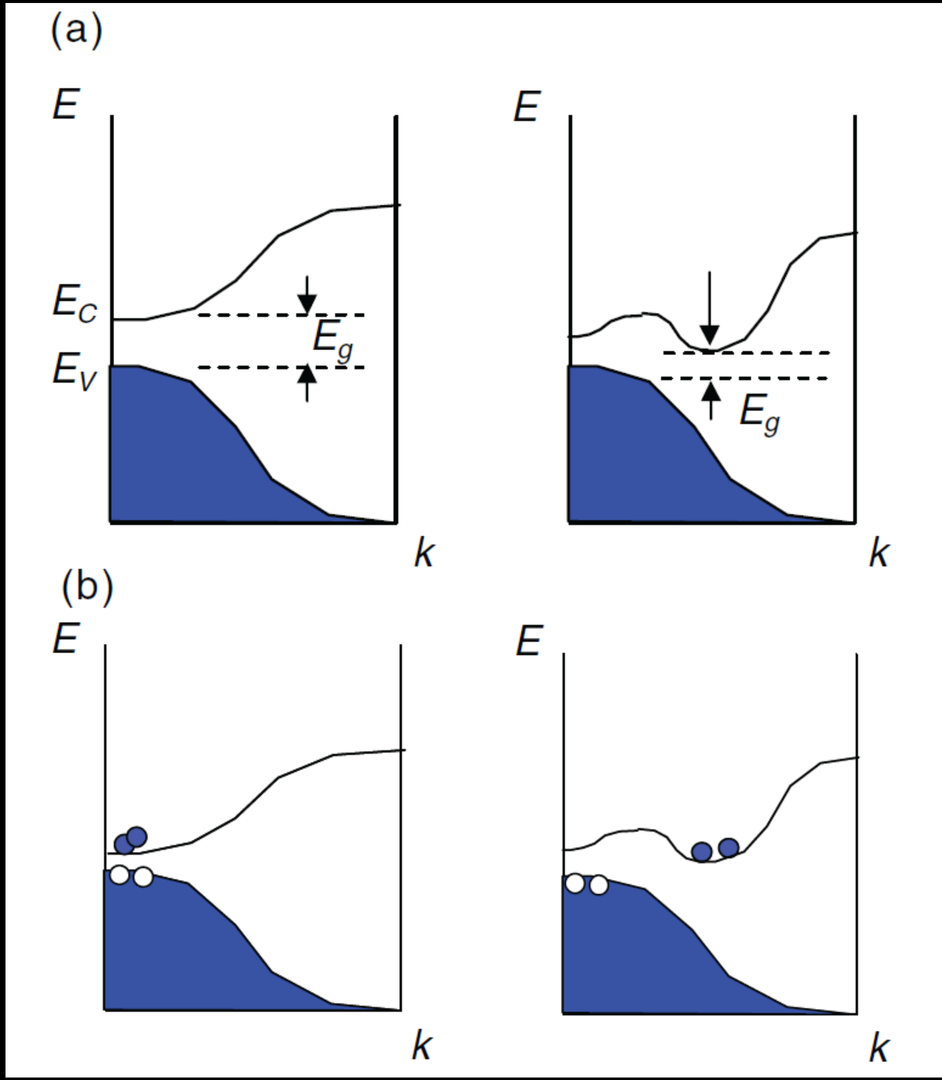
Topological material

A small bandgap of materials, there are direct (Ge) and indirect band gaps(Si). The gap value is in a range of 0.1 to 2 eV (300 K ~ 26 meV). Therefore, they are not very conductive at room temperature!



Type of materials

Metal
Insulator
Semiconductor
Superconductor
Topological material

$$n = \int_{E_C}^{E_{top}} \nu_e(E) f(E, E_F, T) \mathrm{d}E,$$

$$n = \left[2 \left(\frac{2\pi m_e^* k_B T}{\hbar^2} \right)^{3/2} M_C \right] \cdot \exp\left(-\frac{E_C - E_F}{k_B T} \right)$$

$$= N_C(T) \cdot \exp\left(-\frac{E_C - E_F}{k_B T} \right).$$

$$p = \left[2 \left(\frac{2\pi m_h^* k_B T}{\hbar^2} \right)^{3/2} \right] \cdot \exp\left(-\frac{E_F - E_V}{k_B T} \right)$$

$$= N_V(T) \cdot \exp\left(-\frac{E_F - E_V}{k_B T} \right),$$

Type of materials

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$$n_{\mathrm{i}} \equiv \sqrt{np} = \sqrt{N_{\mathrm{C}} N_{\mathrm{V}}} \cdot \exp\left(-\frac{E_{\mathrm{g}}}{2k_{\mathrm{B}}T}\right)$$

$$n_{\mathrm{i}} = 4.9 \times 10^{15} \left(\frac{m_{\mathrm{e}}^{*} m_{\mathrm{h}}^{*}}{m_0^2}\right) M_{\mathrm{C}}^{1/2} T^{3/2} \exp\left(-\frac{E_{\mathrm{g}}}{2k_{\mathrm{B}}T}\right)$$

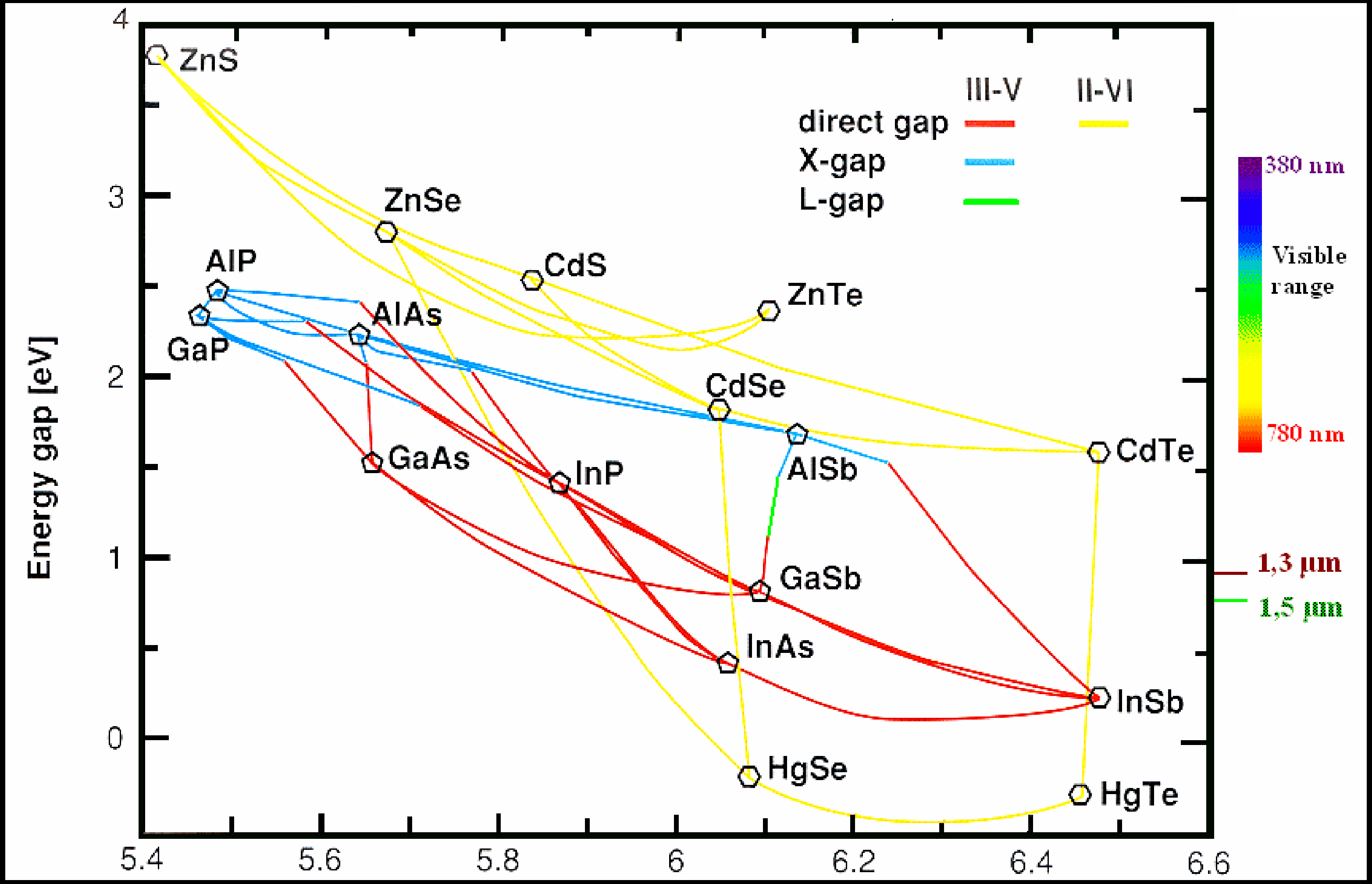
Typical values of carrier density at 300 K:
Si (10^10 cm^{-3}), Ge(10^13 cm^{-3}) and GaAs(10^6 cm^{-3})
While for Cu 10^22 cm-3 (depending on the number of the outer shell electrons)
To increase conductivity of semiconductor, one can add dopants.

Type of materials

https://www.tf.uni-kiel.de/

- Metal
- Insulator
- Semiconductor
- Superconductor
- Topological material

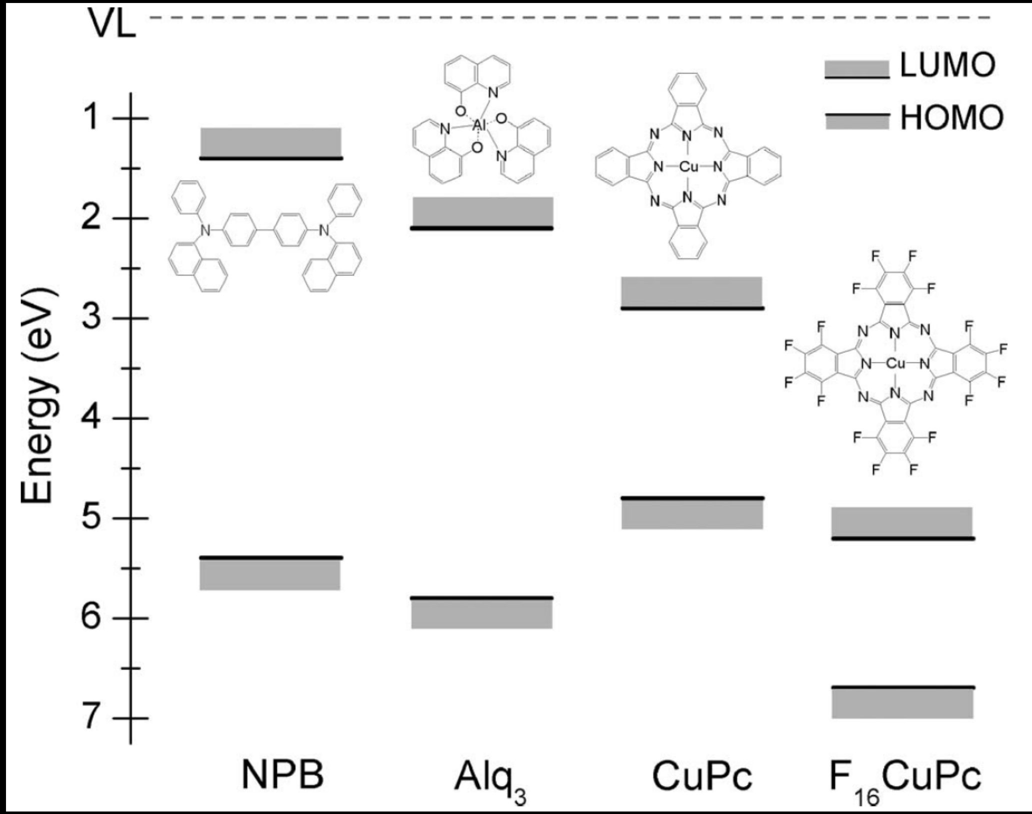
Besides elements, there are a lot of compound semiconductors.



Type of materials

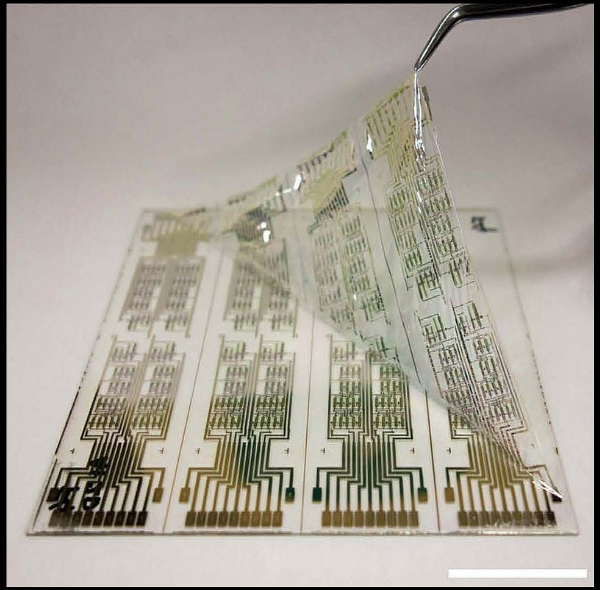
Metal
Insulator
Semiconductor
Superconductor
Topological material

The other is the organic semiconductors which are used in OLED/solar cell applications.



J. X. Tang et al, J. Appl. Phys. 101, (2007)

Wikipedia



Type of materials

Metal
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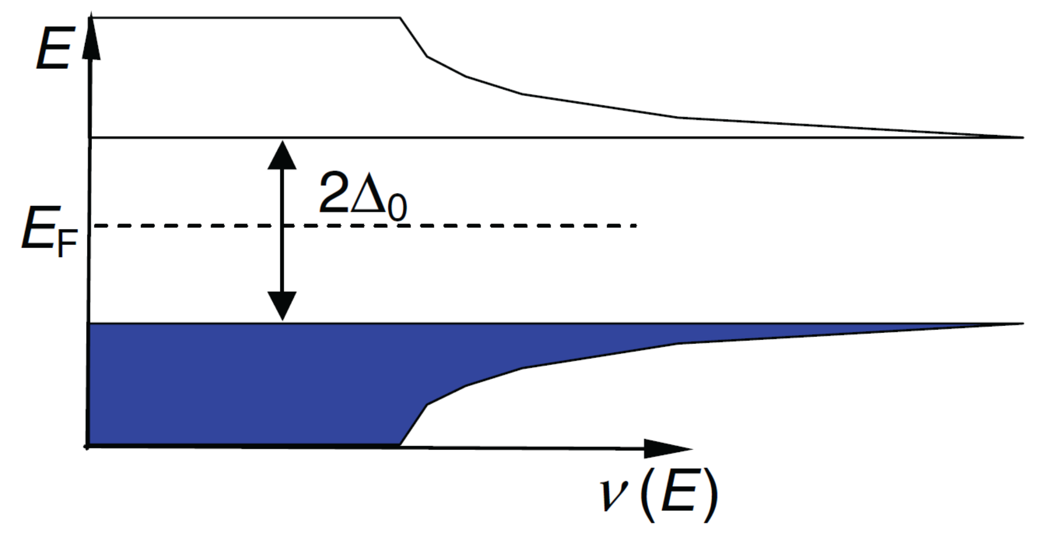
Topological material



Some properties:

1. below Tc, forming a zero DC resistance state
2. cooper pair is two electrons with coupling strength in the order of superconducting gap.
3. Meissner effect, the magnetic field lines exclusion effect

Type of materials

Metal
Insulator
Semiconductor
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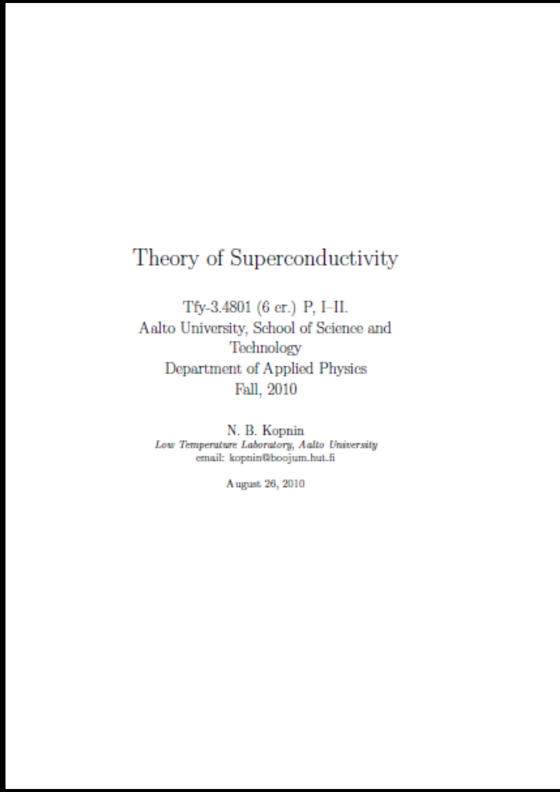
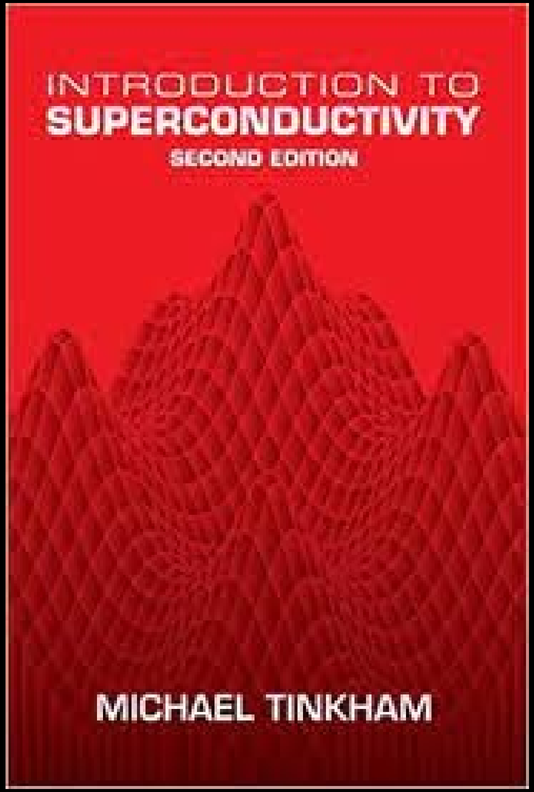
Important theory for SC:
highly recommended books
for superconductivity


Theory of Superconductivity

Tfy-3.4801 (6 cr.) P, I-II.
Aalto University, School of Science and Technology
Department of Applied Physics
Fall, 2010

N. B. Kopnin
Low Temperature Laboratory, Aalto University
email: kopnin@boojum.hut.fi

August 26, 2010



Type of materials

Metal
Insulator
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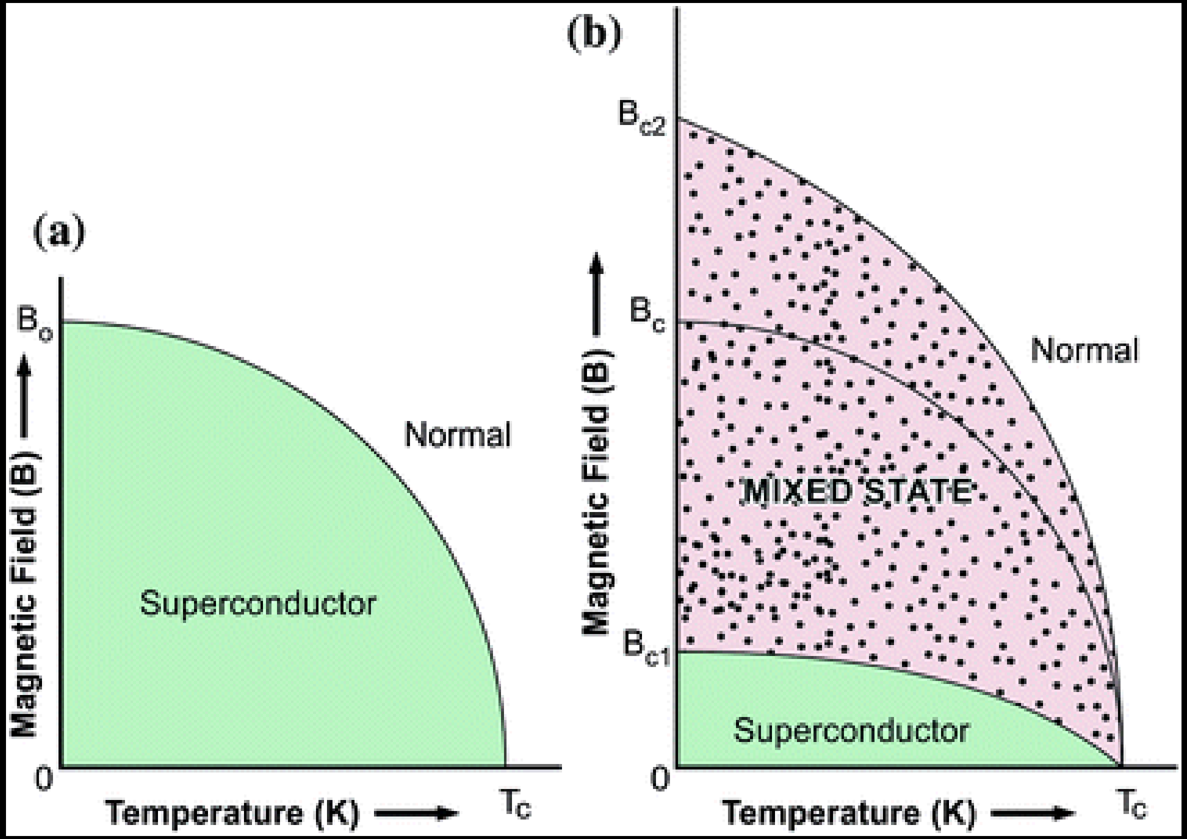
Type of superconductors:

Type I
Type II



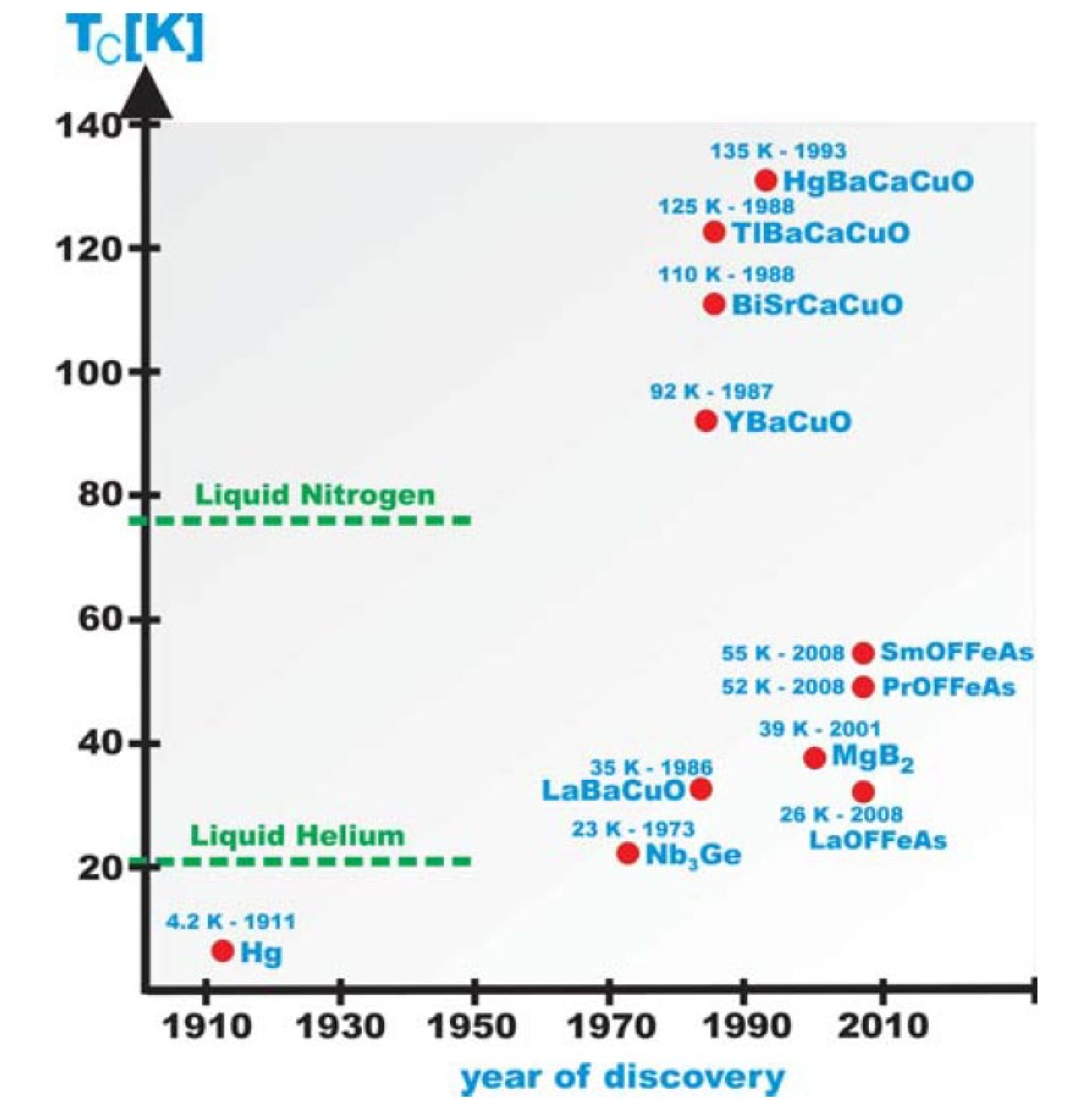
R. G. Sharma

Type of materials

Metal
Insulator
Semiconductor
Superconductor
Topological material

Type of superconductors:

High Tc superconductors



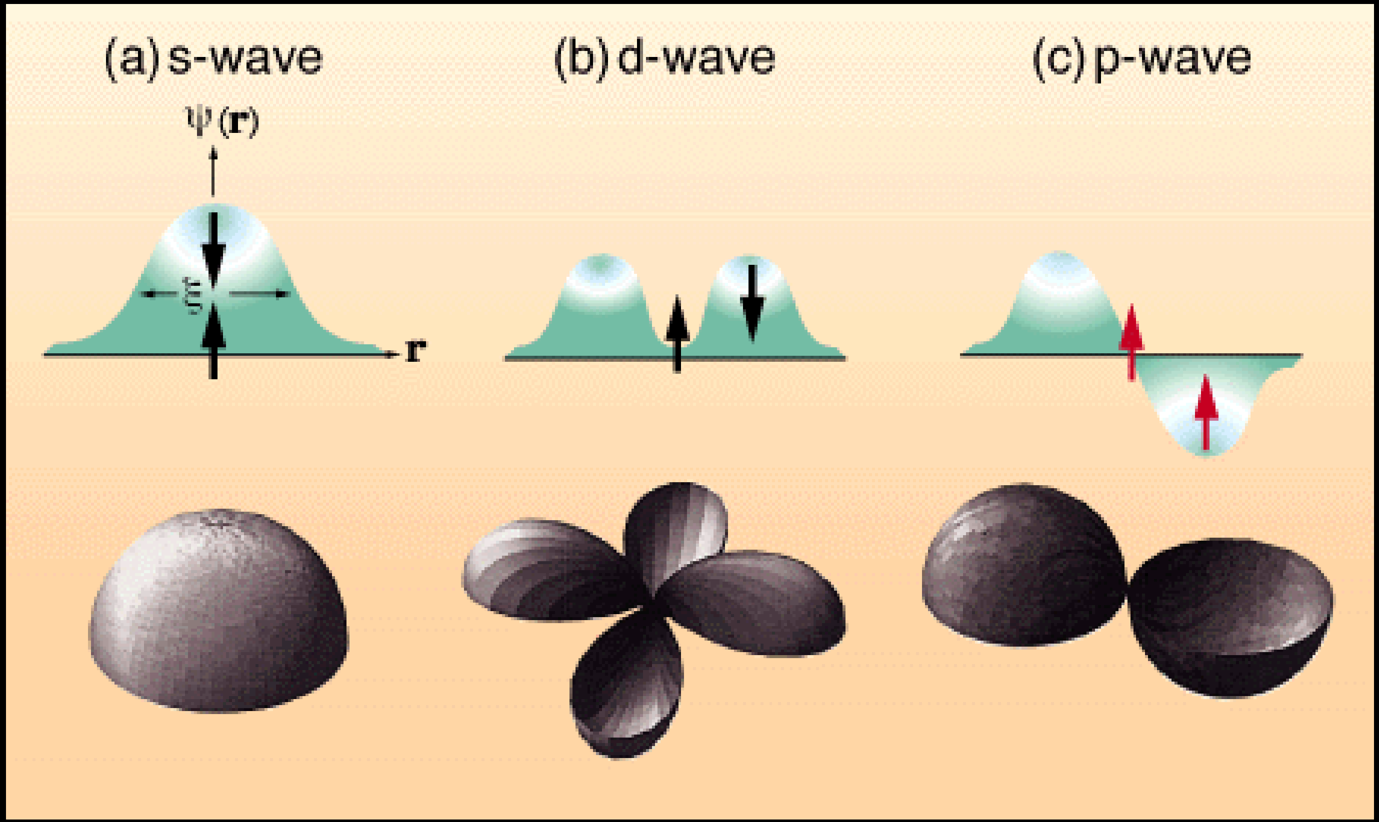
Type of materials

Order parameter symmetry

Metal
Insulator
Semiconductor
Superconductor
Topological material

Type of superconductors:

s-wave
d-wave
p-wave?



https://rdreview.jaea.go.jp/tayu/ACT97E/01/0104.htm

Type of materials

Metal
Insulator
Semiconductor
Superconductor
Topological material

Bogoliubov–de Gennes equations
A theory to describe excitation spectrum for inhomogeneous superconductors

the BCS wave function for the electron (u) and hole (v) in the excitations

$$\Psi^\dagger(\mathbf{r}\uparrow) = \sum_n \left[\gamma_{n\uparrow}^\dagger u_n^*(\mathbf{r}) - \gamma_{n\downarrow} v_n(\mathbf{r})\right]$$

$$\Psi^\dagger(\mathbf{r}\downarrow) = \sum_n \left[\gamma_{n\downarrow}^\dagger u_n^*(\mathbf{r}) + \gamma_{n\uparrow} v_n(\mathbf{r})\right]$$

$$H_{eff} = \int dV \left\{ \sum_s \psi_s^\dagger H_0 \psi_s + \Delta(\mathbf{r}) \psi_\uparrow^\dagger \psi_\downarrow^\dagger + \Delta^*(\mathbf{r}) \psi_\downarrow \psi_\uparrow \right\},$$

$$H_0 = \frac{1}{2m}\left(\mathbf{p} + \frac{e}{c}\mathbf{A}\right)^2 + U(\mathbf{r}) - \mu,$$

Type of materials

Metal
Insulator
Semiconductor
Superconductor
Topological material

Bogoliubov–de Gennes equations
A theory to describe excitation spectrum for inhomogeneous superconductors

$$H_{eff} = \sum_{ns} E_n \gamma_{ns}^{\dagger} \gamma_{ns},$$

Use the relationship:

$$\begin{aligned} \{\gamma_{ns}, \gamma_{n's'}^{\dagger}\} &= \delta_{nn'}\delta_{ss'}, \\ \{\gamma_{ns}, \gamma_{n's'}\} &= 0. \end{aligned}$$

$$\begin{aligned} i\dot{\gamma}_{ns} &= [\gamma_{ns}, H_{eff}] = E_n \gamma_{ns}, \\ i\dot{\gamma}_{ns}^{\dagger} &= [\gamma_{ns}^{\dagger}, H_{eff}] = -E_n \gamma_{ns}^{\dagger}. \end{aligned}$$

$$\begin{aligned} i\dot{\psi}_{\uparrow}(\mathbf{r}) &= [\psi_{\uparrow}(\mathbf{r}), H_{eff}] = H_0 \psi_{\uparrow}(\mathbf{r}) + \Delta(\mathbf{r})\psi_{\downarrow}^{\dagger}(\mathbf{r}), \\ i\dot{\psi}_{\downarrow}(\mathbf{r}) &= [\psi_{\downarrow}(\mathbf{r}), H_{eff}] = H_0 \psi_{\downarrow}(\mathbf{r}) - \Delta(\mathbf{r})\psi_{\uparrow}^{\dagger}(\mathbf{r}). \end{aligned}$$

Type of materials

Metal
Insulator
Semiconductor
Superconductor
Topological material

Bogoliubov–de Gennes equations
A theory to describe excitation spectrum for inhomogeneous superconductors

$$H_0 u_n(\mathbf{r}) + \Delta(\mathbf{r}) v_n(\mathbf{r}) \; = \; E_n u_n(\mathbf{r}),$$

$$H_0^* v_n(\mathbf{r}) - \Delta^*(\mathbf{r}) u_n(\mathbf{r}) \; = \; -E_n v_n(\mathbf{r}).$$

The BdG equation:

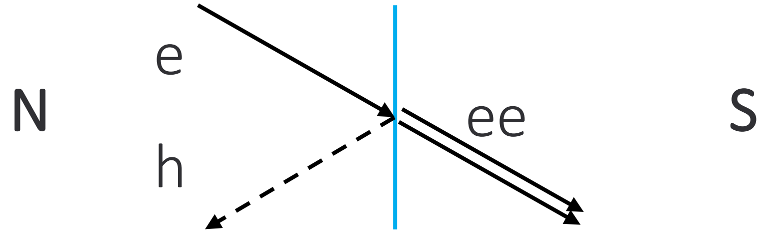
$$\begin{pmatrix} H_0 & \Delta \\ \Delta^* & -H_0^* \end{pmatrix} \begin{pmatrix} u_n \\ v_n \end{pmatrix} = E_n \begin{pmatrix} u_n \\ v_n \end{pmatrix}$$

$$\Delta(\mathbf{r}) = -V_e \langle \psi_\downarrow(\mathbf{r}) \psi_\uparrow(\mathbf{r}) \rangle$$

Note: $\Delta = 0$, we return to ordinary e/h eigenfunction, however, Δ is the function of u and v

Type of materials

Andree reflection:



For the N side $\Delta=0$:

$$\left(\begin{array}{c} U(x) \\ V(x) \end{array}\right)_L = e^{i\lambda_N x}\left(\begin{array}{c} 1 \\ 0 \end{array}\right) + a e^{-i\lambda_N x}\left(\begin{array}{c} 0 \\ 1 \end{array}\right)$$

$$\lambda_N = \frac{\epsilon}{\hbar v_x}$$

Incident electron reflection hole

For the S side(if ε>Δ) :

$$\left(\begin{array}{c} U(x) \\ V(x) \end{array}\right)_R = c e^{i\lambda_S x}\left(\begin{array}{c} U_0 \\ V_0 \end{array}\right) \qquad \lambda_S = \frac{\sqrt{\epsilon^2 - |\Delta|^2}}{\hbar v_x}$$

U_0 and V_0 satisfy the following relationship

$$U_0^2 - V_0^2 = \frac{\sqrt{\epsilon^2 - |\Delta|^2}}{\epsilon}\,,\ U_0 V_0 = \frac{|\Delta|}{2\epsilon}$$

$$a = V_0/U_0\,,\ c = 1/U_0 \qquad |a|^2 + (U_0^2 - V_0^2)|c|^2 = 1$$

if ε<Δ

$$\tilde{U}_0 = \frac{1}{\sqrt{2}}\left(1 + i\frac{\sqrt{|\Delta|^2 - \epsilon^2}}{\epsilon}\right)\,,\ \tilde{V}_0 = \frac{1}{\sqrt{2}}\left(1 - i\frac{\sqrt{|\Delta|^2 - \epsilon^2}}{\epsilon}\right)$$

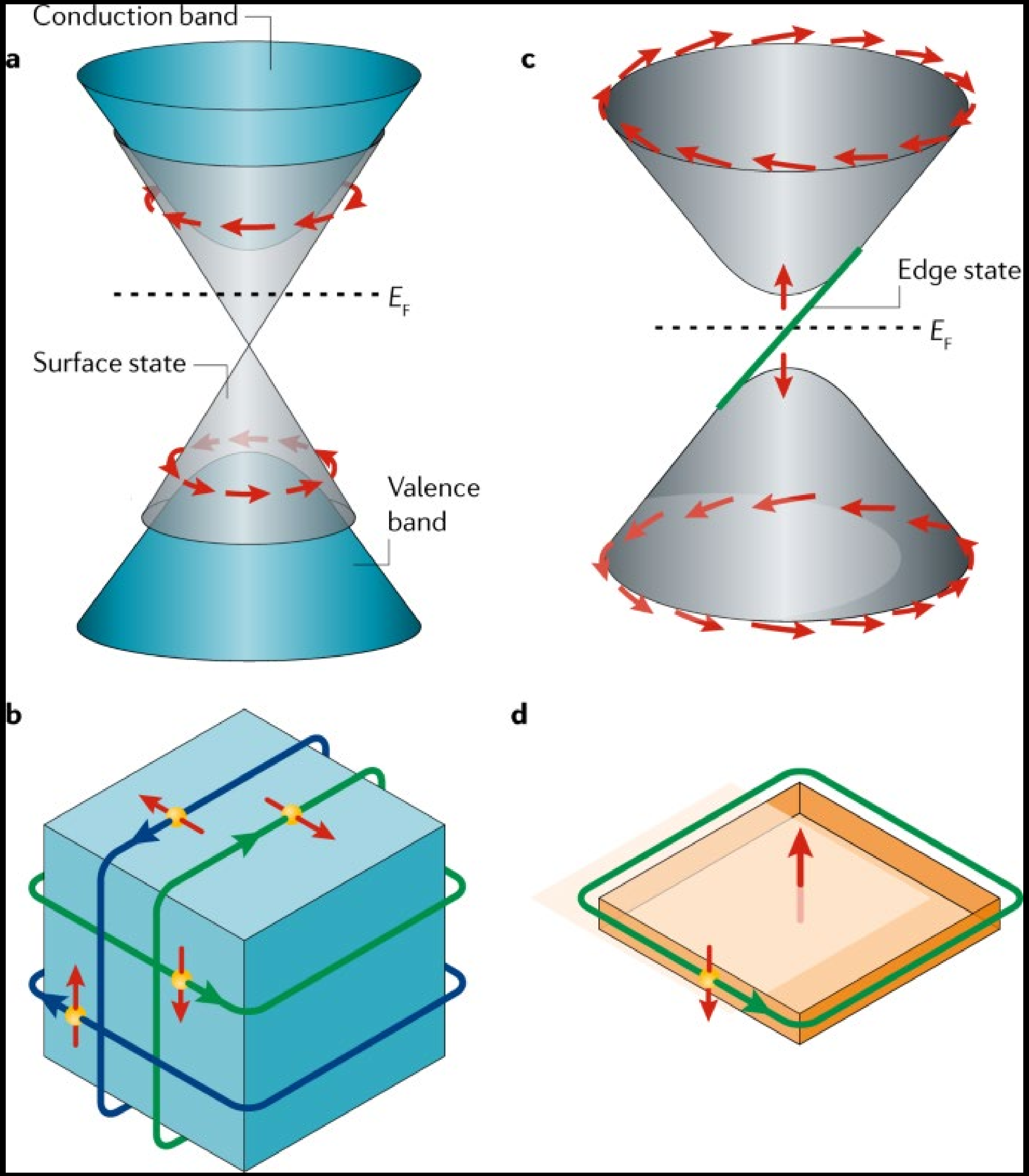
$$a = \tilde{V}_0/\tilde{U}_0\,,\ c = 1/\tilde{U}_0 \qquad |a|^2 = 1$$

Type of materials

Metal
Insulator
Semiconductor
Superconductor
Topological material

Some materials are expected to be boring band insulators, however, it turns out they have very unique band structure at the boundaries.

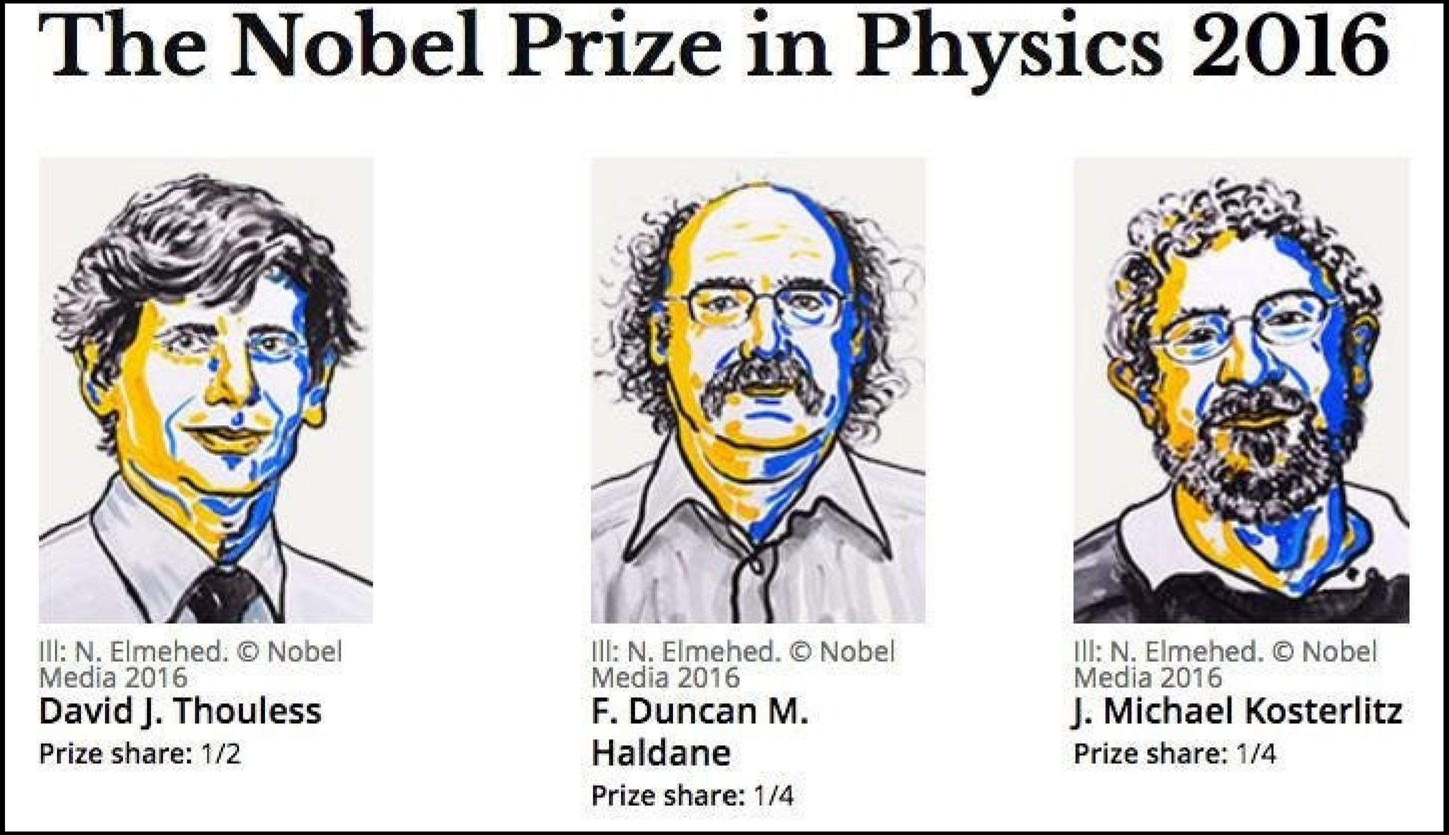
One material family for example is the topological insulators such as Bi_2Se_3, HgTe/CdTe heterostructure etc.



Yoshinori Tokura et al, Nature Review Physics (2019)

Type of materials

Metal
Insulator
Semiconductor
Superconductor
Topological material



Type of materials

Metal
Insulator
Semiconductor
Superconductor
Topological material

Another way to look at the substances is based on the symmetries. The symmetry class is from the Lei group definition and the periodic table shows their symmetry/topological invariant.

T: 0, 1 and -1, P: 0, 1 and -1, C:0

Altland and Zirnbauer (1996)

class	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$	$d=0$	1	2	3
A				$\mathbb{Z}$		$\mathbb{Z}$	
AI			1	$\mathbb{Z}$			
AII			-1	$2\mathbb{Z}$		$\mathbb{Z}_2$	$\mathbb{Z}_2$
AIII	1				$\mathbb{Z}$		$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}$		
C		-1				$2\mathbb{Z}$	
CI	1	-1	1				$2\mathbb{Z}$
CII	1	-1	-1		$2\mathbb{Z}$		$\mathbb{Z}_2$
D		1		$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	
DIII	1	1	-1		$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

https://topocondmat.org/w8_general/classification.html
https://math.ucr.edu/home/baez/tenfold.html

Type of materials

Metal
Insulator
Semiconductor
Superconductor
Topological material

- Metal with broken time-reversal symmetry: “Unitary” class A
- Spinless time-reversal invariant superconductor: class BDI
- Spin-1/2 superconductor with no physical symmetries: Class D

Altland and Zirnbauer (1996)

class	$\mathcal{C}$	$\mathcal{P}$	$\mathcal{T}$	$d=0$	1	2	3
A				$\mathbb{Z}$		$\mathbb{Z}$	
AI			1	$\mathbb{Z}$			
AII			-1	$2\mathbb{Z}$		$\mathbb{Z}_2$	$\mathbb{Z}_2$
AIII	1				$\mathbb{Z}$		$\mathbb{Z}$
BDI	1	1	1	$\mathbb{Z}_2$	$\mathbb{Z}$		
C		-1				$2\mathbb{Z}$	
CI	1	-1	1				$2\mathbb{Z}$
CII	1	-1	-1		$2\mathbb{Z}$		$\mathbb{Z}_2$
D		1		$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$	
DIII	1	1	-1		$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$

https://topocondmat.org/w8_general/classification.html
https://math.ucr.edu/home/baez/tenfold.html

Course Syllabus

Date	Topic	Instructor
2/19	Introduction and review	Chung-Ting Ke
2/26	Charge transport review and nanoelectronics	Chung-Ting Ke
3/5	Carbon-based nanomaterials	Chung-Ting Ke
3/12	Superconductivity	Tien-Ming Chuang
3/19	Midterm exam I	Chung-Ting Ke
3/26	Quantum transport	Chen-Hsuan Hsu
4/2	Superconducting theory and devices	Chung-Ting Ke
4/9	Magnetism and magnetoelectronics (1)	Shang Fan Lee
4/16	Magnetism and magnetoelectronics (2)	Shang Fan Lee
4/23	2D materials	Chung-Ting Ke
4/30	Thermoelectricity in nanoscale systems	Ou Min-Nan
5/07	Nano-optics	Yu-Chieh Wen
5/14	Midterm II	Chung-Ting Ke
5/21	Topological materials 1	Tien-Ming Chuang
5/28	Quantum Computation	Chung-Ting Ke
6/4	Study group oral presentation (I)	Chung-Ting Ke
6/11	Study group oral presentation (II)	Chung-Ting Ke

Text book and Evaluation

Textbook:

Nanostructures and nanotechnology by Douglas Natelson, Cambridge University Press (2015)
eBook available (https://doi.org/10.1017/CBO9781139025485) if accessed from AS domain.

Evaluation:

Quiz and attendance 30%

Midterm I 20%

Midterm II 20%

Final presentation 40%

Charge transport and nanoelectronics II

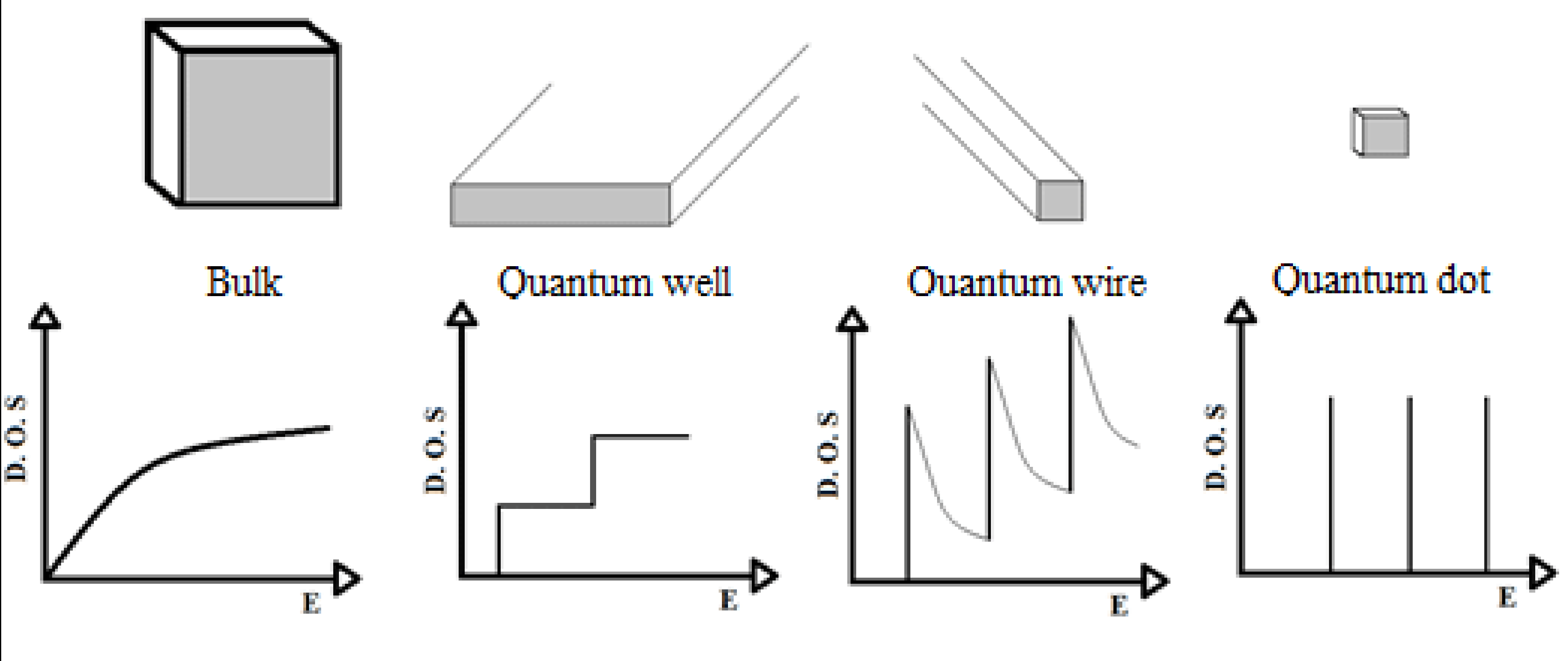
TIGP Advanced Nanotechnology

Chung-Ting Ke

02/26/2025

Electron transport

Different dimensions



Teich, M.C. and B. Saleh, Fundamentals of photonics (1991)

Electron transport

Length scales

dimensions	coordinate region	spacing in k	Fermi region	number of electron, electron density, Fermi wave length N=Fermi region/spacing in k, $\lambda_F=2\pi/k_F$		
1	**Length L**	$2\pi/L$	$2k_F$	$N=\frac{4k_F}{2\pi/L}$	$n=\frac{N}{L}=\frac{4k_F}{2\pi}$	$\lambda_F=\frac{4}{n}$
2	**Area L^2**	$(2\pi/L)^2$	πk_F^2	$N=\frac{2\pi k_F^2}{(2\pi/L)^2}$	$n=\frac{N}{L^2}=\frac{k_F^2}{2\pi}$	$\lambda_F=\left(\frac{2\pi}{n}\right)^{1/2}$
3	**Volume L^3**	$(2\pi/L)^3$	$\frac{4}{3}\pi k_F^3$	$N=\frac{2(4\pi k_F^3/3)}{(2\pi/L)^3}$	$n=\frac{N}{L^3}=\frac{k_F^3}{3\pi^2}$	$\lambda_F=2\left(\frac{\pi}{3n}\right)^{1/3}$

Heinzel, Ch 5.2, p.139

Electron transport

Mean Free Path

Drude model: $J = \sigma E = \frac{ne^2\tau}{m} E$

$$J = nev$$

Drift velocity

Mean scattering time

$$\tau = \frac{m}{e}\frac{v}{E} = \frac{m\mu}{e}$$

Electron mobility

$$l_e = v_F \tau = \frac{\hbar k_F}{m}\frac{m\mu}{e} = \frac{\hbar\mu}{e} k_F$$

Dimension	Fermi wavelength	Mean free path
1	$\lambda_F = \frac{4}{n}$	$l_e = \frac{\hbar\mu}{e}\frac{n\pi}{2}$
2	$\lambda_F = \left(\frac{2\pi}{n}\right)^{1/2}$	$l_e = \frac{\hbar\mu}{e}\sqrt{2\pi n}$
3	$\lambda_F = 2\left(\frac{\pi}{3n}\right)^{1/3}$	$l_e = \frac{\hbar\mu}{e}(3n)^{1/3}\pi^{2/3}$

Heinzel, Ch 5.2, p.140

Electron transport

Diffusion coefficient

Diffusive current flow: *J=-D∇ n (due to the density gradient), D is the diffusion constant*

Einstein relation
(Brownian motion):

$$D = \frac{\mu k_B T}{e}$$

$$\sigma = \frac{ne^2\tau}{m} = ne\mu \Rightarrow \mu = \frac{e\tau}{m}$$

$$l_e = v_F \tau$$

Dimension	Equipartition theorem	Diffusion constant
1	$\frac{1}{2}mv_F^2 = \frac{1}{2}k_BT$	$\mathrm{D} = \frac{l_e v_F}{\mathrm{d}}, \mathrm{d} = 1,2,3$
2	$\frac{1}{2}mv_F^2 = k_BT$	
3	$\frac{1}{2}mv_F^2 = \frac{3}{2}k_BT$	

Heinzel, Ch 5.3, p.141

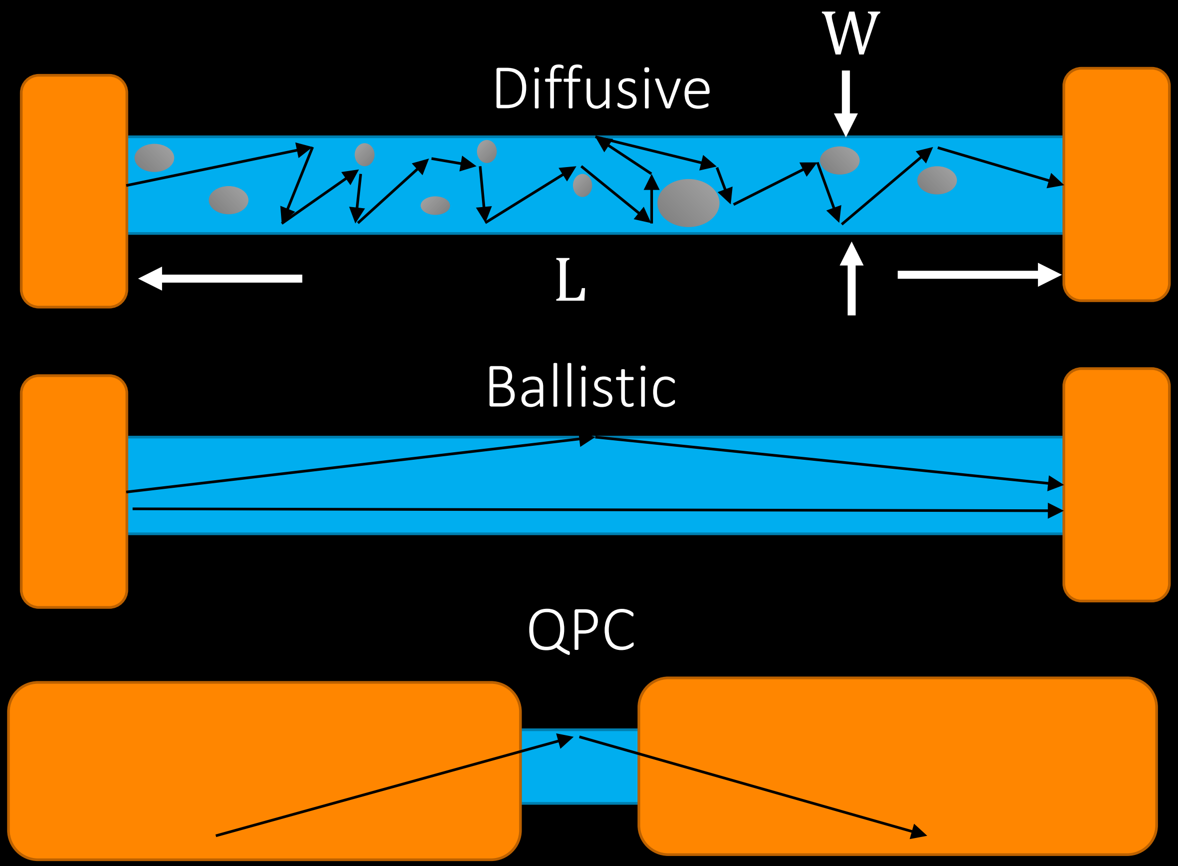
Electron transport

Diffusion vs. Ballistic $l_e = v_F \tau$

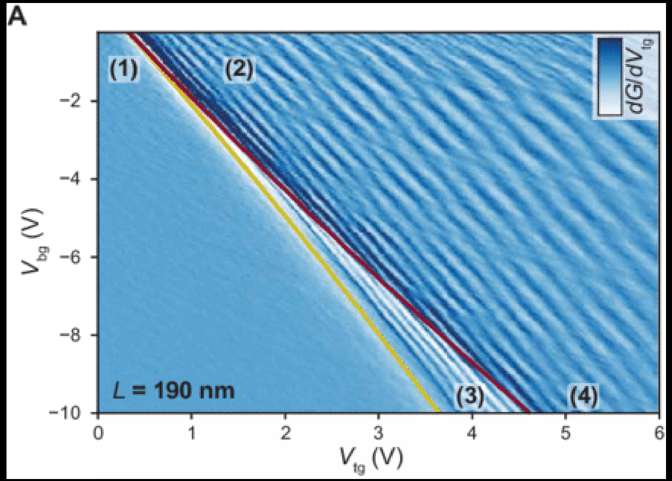
$l_e \ll$ W and L

$l_e >$ W and L

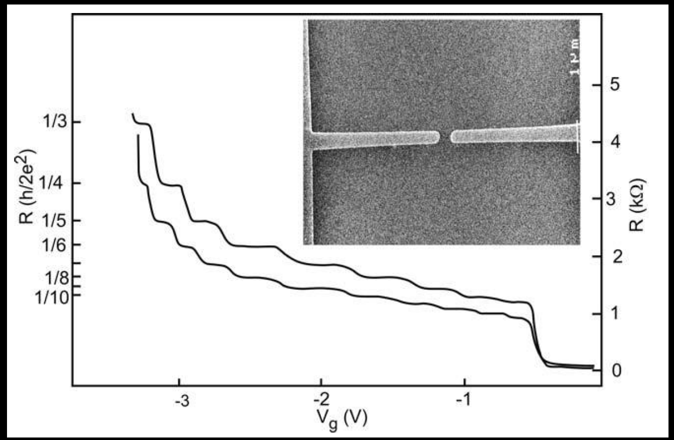
$l_e \gg$ W and L



Fabry-Perot oscillations

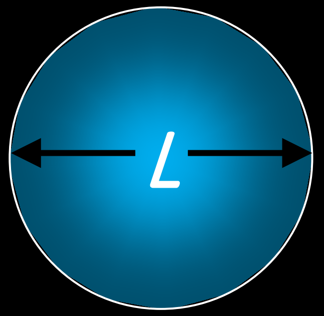


Peter Rickhaus et al. Science Advances (2020)



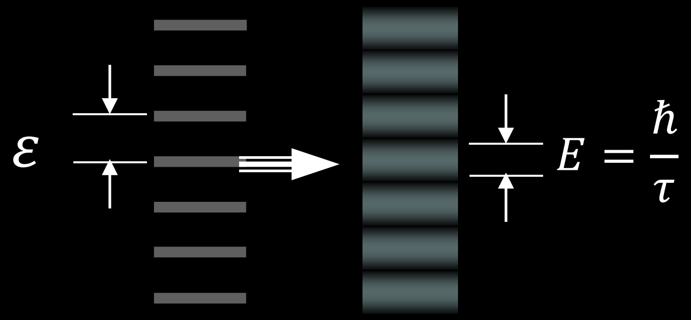
Heinzel

Electron transport



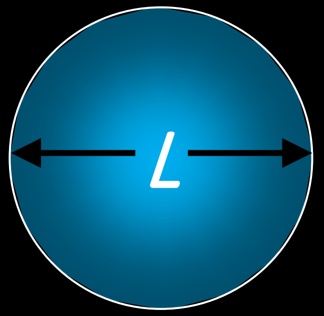
Thouless Energy: E_{Th}

Here, one can consider a diffusive system with size L, the quantum levels have a spacing of $\varepsilon = 1/N(\varepsilon_F) \cdot L^d$.



In the diffusive system, another characteristic energy scale is the Thouless energy $E_{Th} = \frac{\hbar D}{L^2}$, it can be considered as the traveling time (depending on the diffusion time) through the system.

Electron transport



Thouless Energy: E_{Th}

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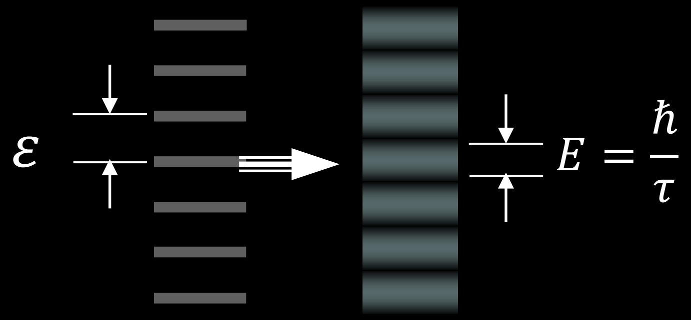


In the diffusive system, another characteristic energy scale is the Thouless energy $E_{Th} = \frac{\hbar D}{L^2}$, it can be considered as the traveling time (depending on the diffusion time) through the system.

Conductance (Einstein relation) $G = \sigma L^{d-2} = \frac{e^2 N(\varepsilon_F) L^d D}{L^2} = \sigma_0 g(L)$ where $\sigma_0 = \frac{e^2}{\hbar}$

and $g(L) = \frac{\hbar D / L^2}{1/N(\varepsilon_F) L^d} = \frac{E_{Th}}{\varepsilon}$ (dimensionless Thouless conductance)

$g(L) < 1$ (R> $\frac{\hbar}{e^2}$) $\Rightarrow$ localized state

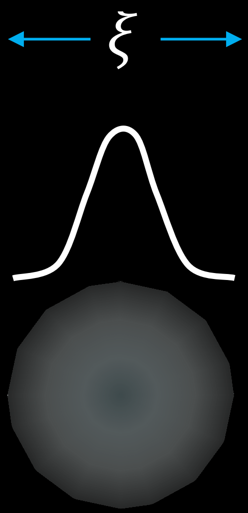
$g(L) => 1$ (R< $\frac{\hbar}{e^2}$) $\Rightarrow$ extended state

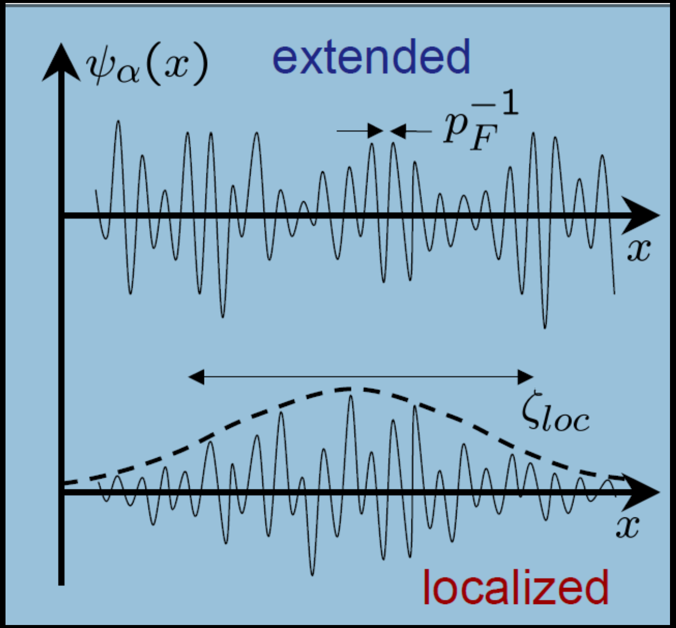
Electron transport

Anderson localization

$g(L) < 1 \Rightarrow$ localized state $\quad G \propto e^{-L/\xi}$

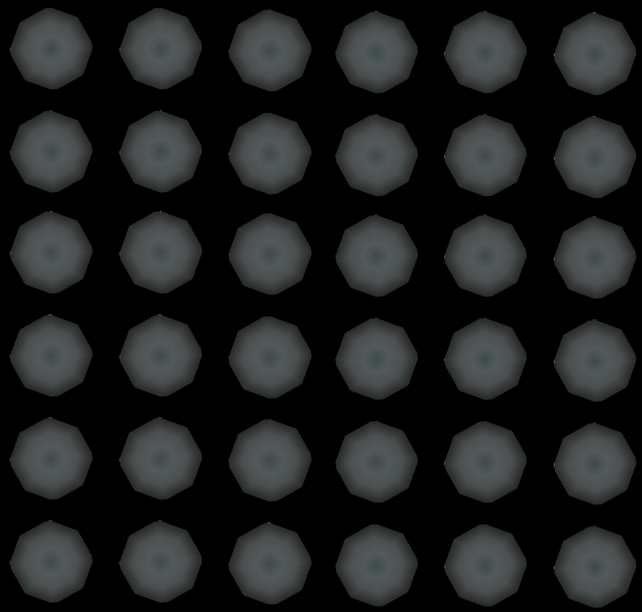
$g(L) > 1 \Rightarrow$ extended state $\quad G \propto L^{d-2}$





Boris Altshuler lecture

Anderson model



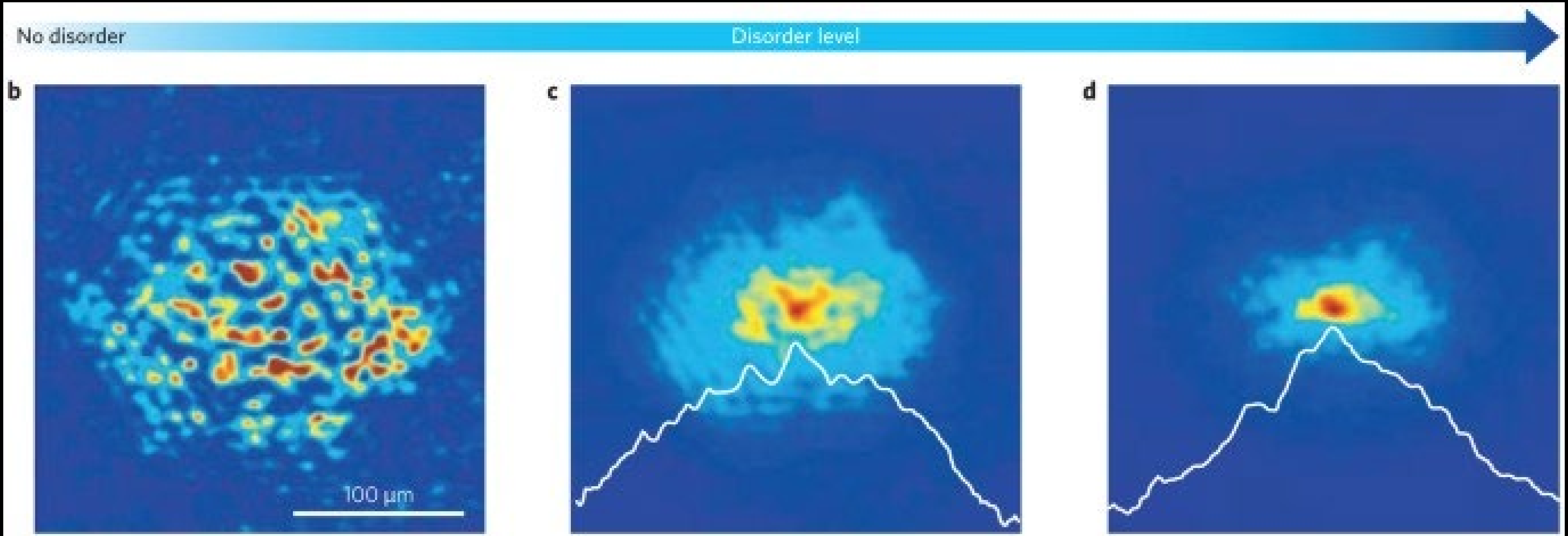
Lattice is considered with the Tight binding
Random on-site energy on the lattice
Only the nearest-neighbor hopping.

As the result, the will be a critical value, Ic
I<Ic (localized, insulator)
I>Ic (extended, metal)

Electron transport

Anderson localization

Ballistic diffusive localization



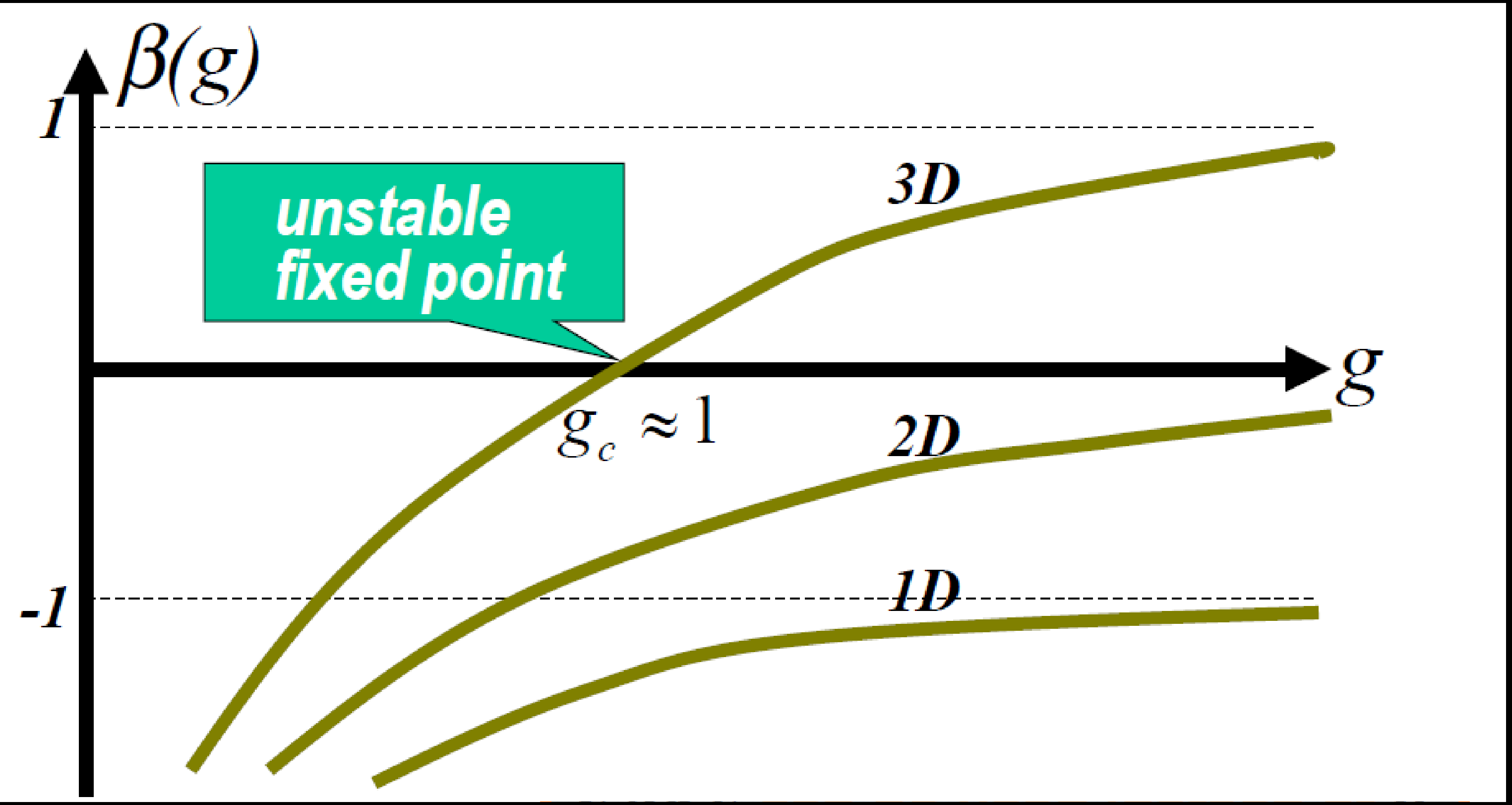
Mordechai Segev et al, Nature Photonics (2013)

Electron transport

$$\frac{d\log(g)}{d\log(L)} = \beta(g)$$

Anderson localization

Scaling theory of localization, dimensionless g is function of L

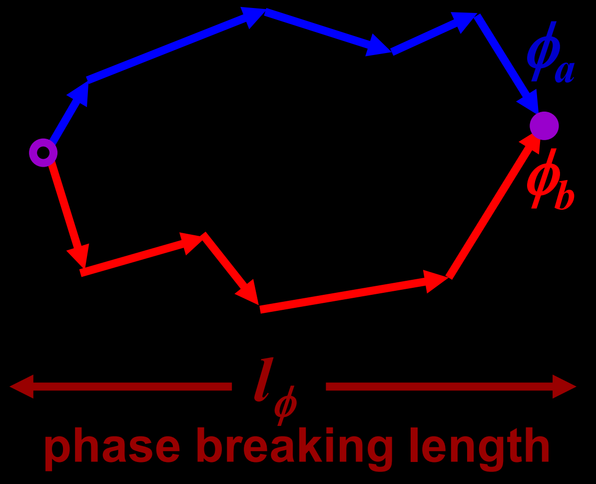


It's universal and material independent. It only matters with global symmetry.

Metal-insulator transition in 3D case but all states in 2D and 1D are localized.

Electron transport

Phase Coherence Length



$$L_\varphi = V_F \tau_\varphi \qquad L_\varphi = \sqrt{D\tau_\varphi}$$

Phase-breaking :
Collisions between electrons, electron-phonon scattering
Collisions with impurities that have an internal degree of freedom

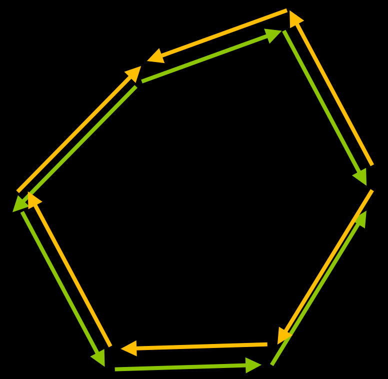
Dimension	Note: (1) Dimensionality in terms of l_ϕ (2) $l_\phi > l$ Localization correction δσ to the Drude conductivity
1	$\delta\sigma_{1D} = -\dfrac{e^2 L_\varphi}{\pi\hbar}$
2	$\delta\sigma_{2D} = -\dfrac{e^2}{2\pi^2\hbar}\ln\dfrac{L_\varphi}{l}$
3	$\delta\sigma_{3D} = -\dfrac{e^2}{2\pi^2\hbar}\left[\dfrac{1}{l} - \dfrac{1}{L_\varphi}\right]$

M. Papper, Review Lecture: Quantum Processes in Semiconductor Structures
Vol. 420, No. 1858 (Nov. 8, 1988), pp. 1-19

Heinzel, Ch 5.4, p.143

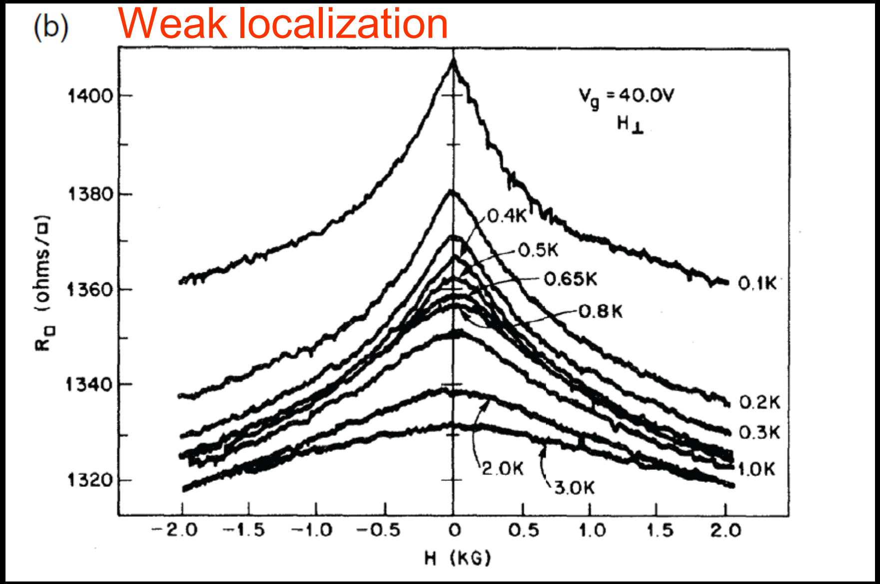
Electron transport

Weak localization (disorder system)



O: $\varphi_a = \varphi_b$

$$W_{a\to b} = \left|\sum_i A_i\right|^2 = \underbrace{\sum_i |A_i|^2}_{\text{Classical diffusion}} + \underbrace{\sum_{i\neq j} A_i A_j^*}_{\text{Quantum interference}}$$



In zero field, B=0, $A_+ = A_- = A$

Probability for back-scattering:

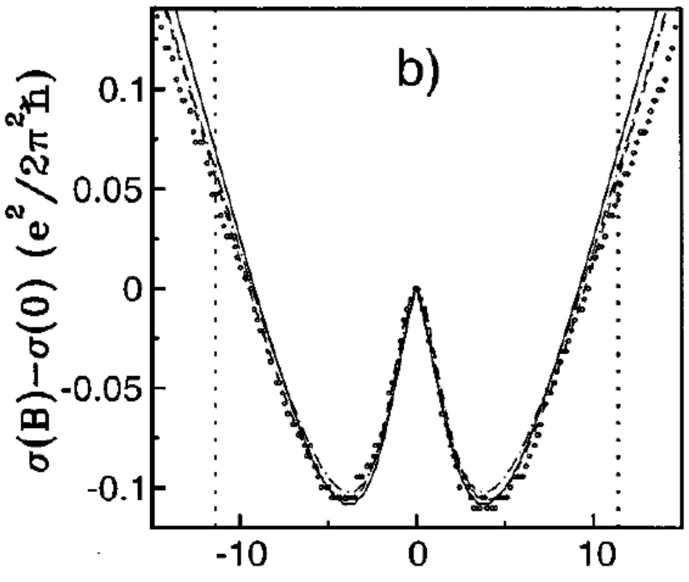
$$W_{O\leftrightarrow O} = |A_+ + A_-|^2 = 4|A|^2$$

The probability of returning to the same position becomes higher. This means lower conductance thus higher resistance.

Electron transport

Weak anti-localization

A disordered system is under the influence of spin-orbit interaction which diminishes the localization due to spin processes. Now, the conductance is enhanced instead lower. Under a certain scenario, the conductance correction:



W. Knap et al. PRB (1996)

Only cubic term

$$\Delta\sigma(B)-\Delta\sigma(0)=\frac{e^2}{2\pi^2\hbar}\left\{\Psi\left(\frac{1}{2}+\frac{H_\varphi}{B}+\frac{H_{SO}}{B}\right)+\frac{1}{2}\Psi\left(\frac{1}{2}+\frac{H_\varphi}{B}+2\frac{H_{SO}}{B}\right)-\frac{1}{2}\Psi\left(\frac{1}{2}+\frac{H_\varphi}{B}\right)-\ln\frac{H_\varphi+H_{SO}}{B}-\frac{1}{2}\ln\frac{H_\varphi+2H_{SO}}{B}+\frac{1}{2}\ln\frac{H_\varphi}{B}\right\}.$$

Only linear term

$$\Delta\sigma(B)-\Delta\sigma(0)=\frac{e^2}{2\pi^2\hbar}\left\{\Psi\left(\frac{1}{2}+\frac{H_\varphi}{B}\right)-\ln\frac{H_\varphi}{B}\right\}$$

This allows us to study the spin-orbit effect of the system.

Electron transport

Quantum phase under a field

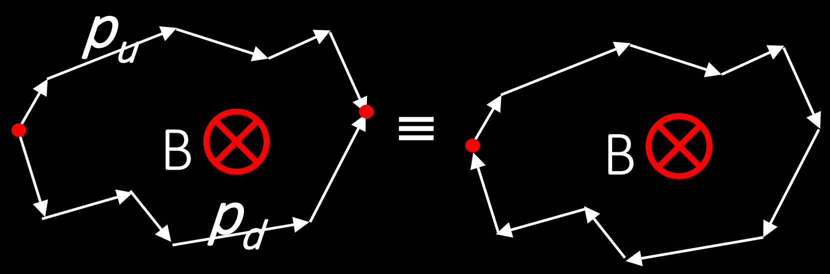
In a field B=∇×A

Canonical momentum

$$p = mv - eA$$

Magnetic field: break the time reversal symmetry

electrons pick a phase φ when traveling along with a path P



phase difference between two paths with the same ends

$$\delta\varphi = \frac{1}{\hbar}\int_P^Q p_u \cdot dl_1 - \frac{1}{\hbar}\int_P^Q p_d \cdot dl_2 = \boxed{\frac{1}{\hbar}\int_P^Q p_u \cdot dl_1 + \frac{1}{\hbar}\int_Q^P p_d \cdot dl_2} = \frac{1}{\hbar}\oint p \cdot dl$$

$$= \frac{e}{\hbar}\int (\nabla \times A) \cdot dS = \frac{e}{\hbar}\int B dS = 2\pi \frac{B \cdot S}{h/e}$$

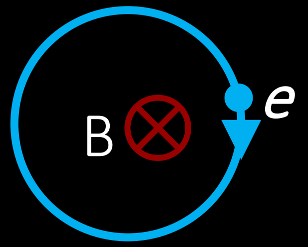
acquired phase around a loop

Electron transport

In a magnetic field

Magnetic length:

Cyclotron motion



Cyclotron frequency $$F = evB = mr_c\omega_c^2$$

Cyclotron radius $$r_c\omega_c = v_F \longrightarrow r_c = \frac{v_F}{\omega_C} = \frac{mv_F}{eB} = \frac{\hbar k_F}{eB}$$

l_B = magnetic length

flux quantization $$B \cdot \pi r_c^2 = n\Phi_0 = \frac{nh}{e} \longrightarrow r_c = \sqrt{\frac{2n\hbar}{eB}} \equiv \sqrt{2n}\, l_B$$

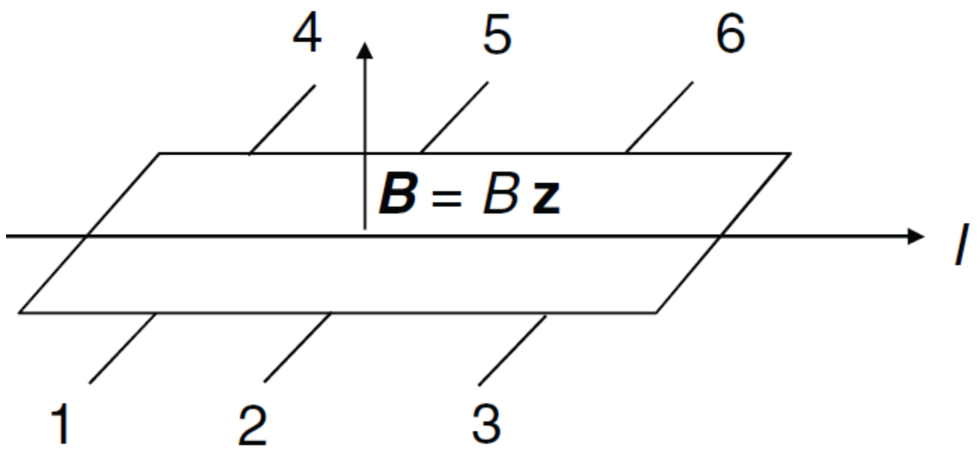
$$2\pi l_B^2 \cdot B = \Phi_0$$

$$B = \frac{\hbar^2 k_F^2}{2ne\hbar} = \frac{mE_F}{ne\hbar} \longrightarrow$$ Quantizing magnetic field

Heinzel, Ch 5.4, p.143

Electron transport

Quantum Hall effect



$$\frac{m\mathbf{v}_\mathrm{d}}{\tau} = q[\mathbf{E} + \mathbf{v}_\mathrm{d} \times \mathbf{B}].$$

$$\begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} m/q^2 n_{2\mathrm{d}}\tau & -m\mu B/q^2 n_{2\mathrm{d}}\tau \\ m\mu B/q^2 n_{2\mathrm{d}}\tau & m/q^2 n_{2\mathrm{d}}\tau \end{pmatrix} \begin{pmatrix} J_x \\ J_y \end{pmatrix}$$

Use the following conditions:

$$\rho_{xx} = \rho_{yy}$$

$$\rho_{yx} = -\rho_{xy} = B/|q|n_{2\mathrm{d}}$$

$$V_x = E_x L,\ V_\mathrm{H} = E_y w$$

One can obtain:

$$n_{2\mathrm{d}} = \frac{I/|q|}{\mathrm{d}V_\mathrm{H}/\mathrm{d}B},$$ Carrier density

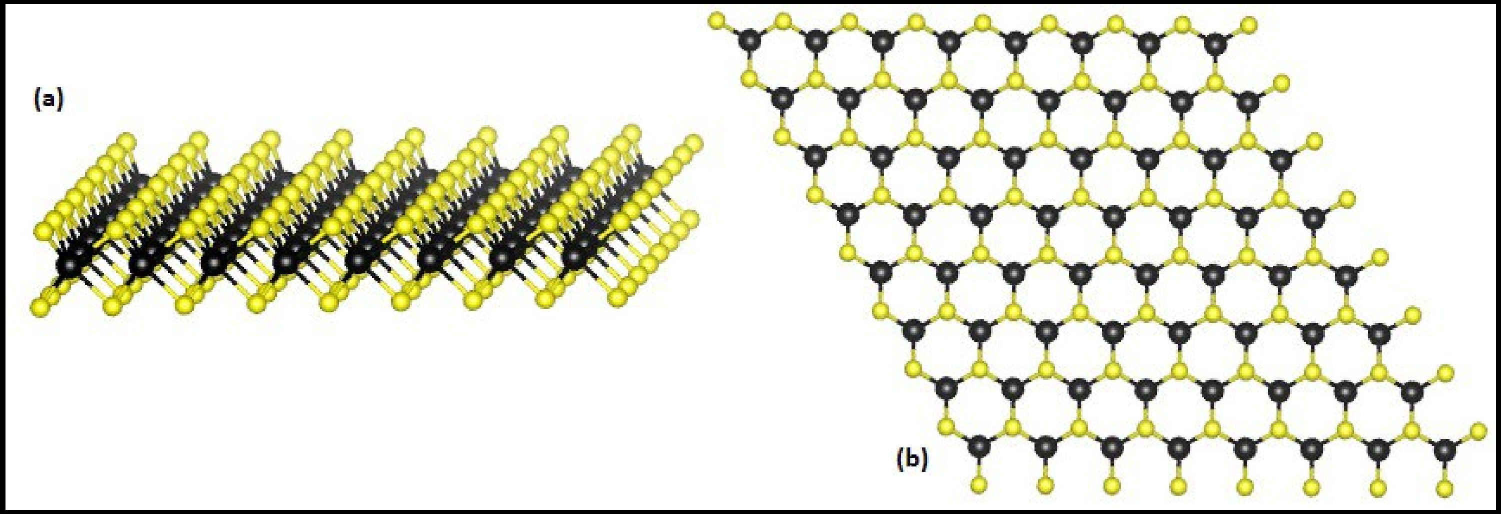
$$\mu = \frac{I/|q|}{n_{2\mathrm{d}} V_x w/L}.$$ Mobility

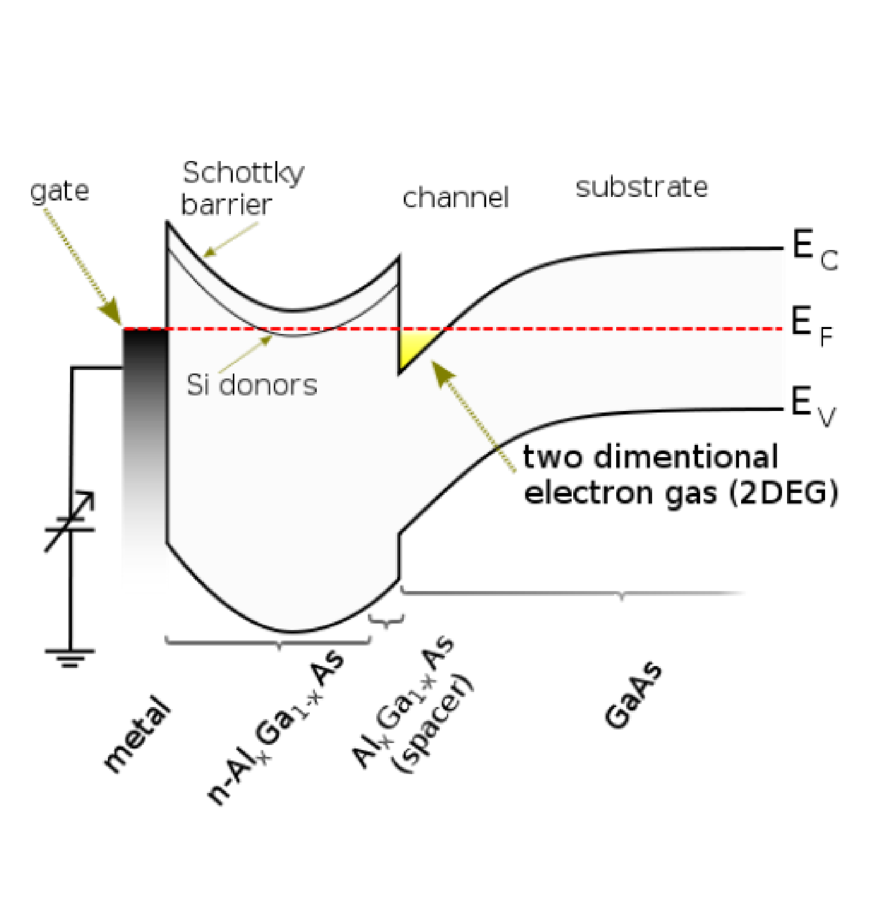
Electron transport

2D electron gas

In a quantum well type structure, the potential well confines electrons/holes in a 2D plane. Charges only allow moving in the X-Y plane

Another type is real 2d material such as graphene or transition metal dichalcogenide





wiki

Electron transport

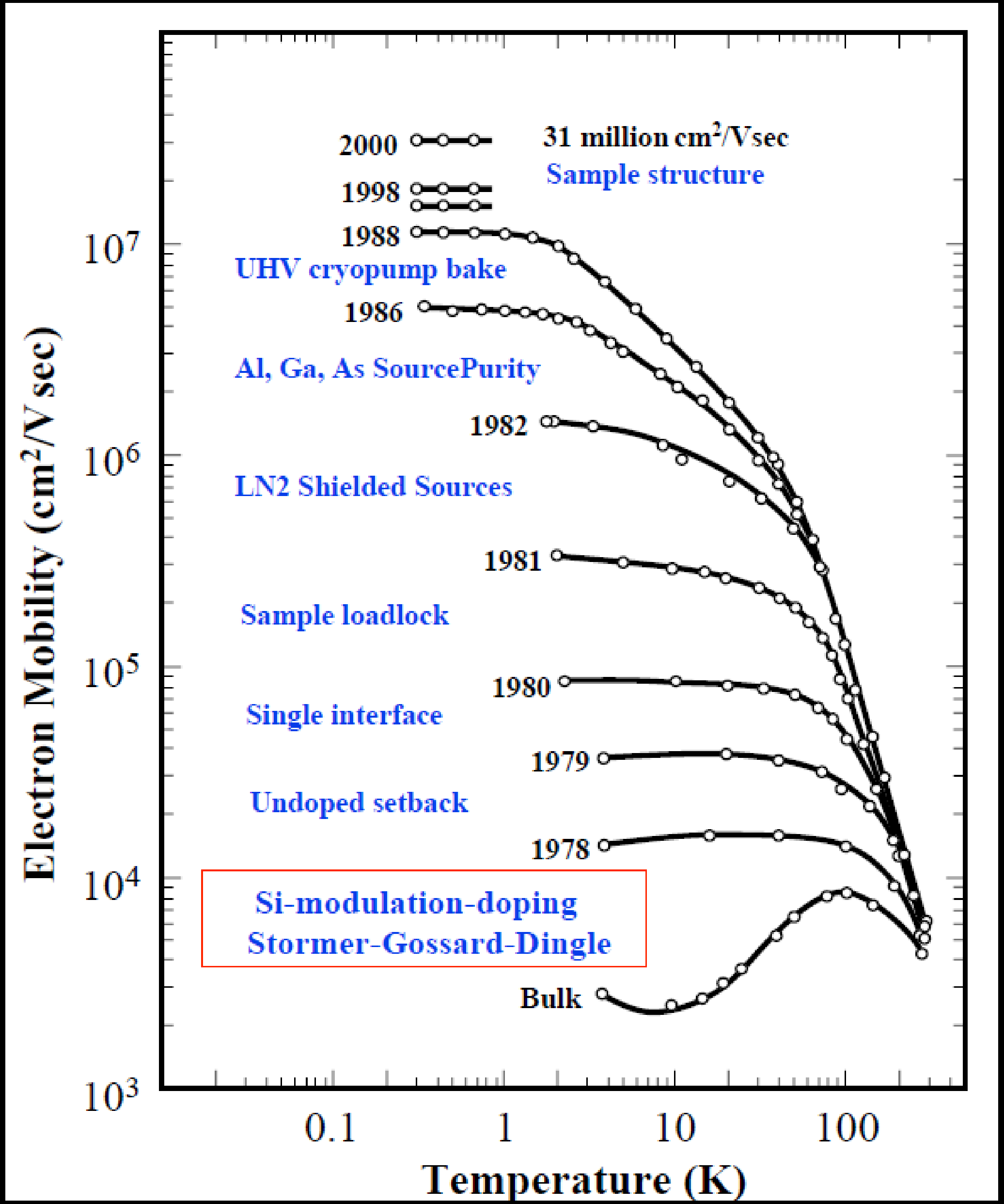
Quantum Hall effect

Developing high mobility samples has a long history, GaAs quantum wells.

It took almost three decades to boost the mobility from 10k to 10 M cm^2/Vs along with this development amazing physics was discovered.

Quantum Hall effect, Nobel prize 1985

Fractional quantum Hall effect, Nobel prize 1998

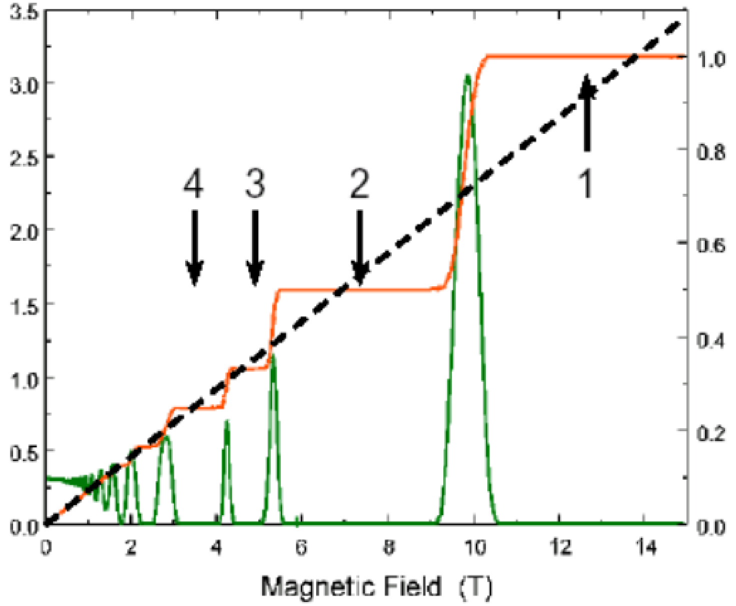


Electron transport

Quantum Hall effect

Landau quantization(David Tong, The Quantum Hall effect):

$$H = \frac{1}{2m}(\mathbf{p} + e\mathbf{A})^2 \qquad \nabla \times \mathbf{A} = B\hat{\mathbf{z}}$$



$$[x_i, p_j] = i\hbar\delta_{ij} \quad \text{with} \quad [x_i, x_j] = [p_i, p_j] = 0$$

The mechanical momentum $\boldsymbol{\pi} = \mathbf{p} + e\mathbf{A} = m\dot{\mathbf{x}}$

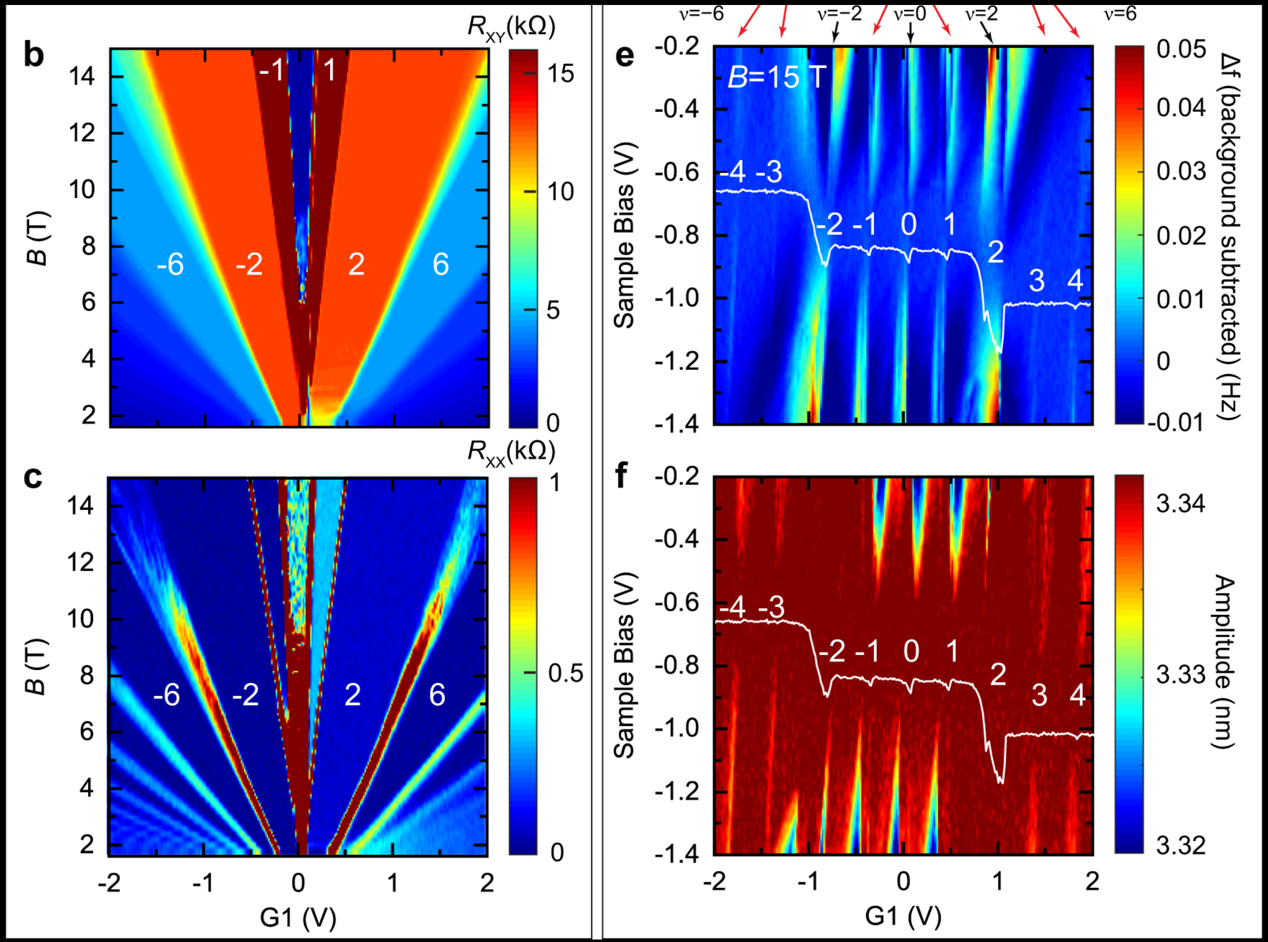
$$\{m\dot{x}_i, m\dot{x}_j\} = \{p_i + eA_i, p_j + eA_j\} = -e\left(\frac{\partial A_j}{\partial x^i} - \frac{\partial A_i}{\partial x^j}\right) = -e\epsilon_{ijk}B_k$$

$$a = \frac{1}{\sqrt{2e\hbar B}}(\pi_x - i\pi_y) \quad \text{and} \quad a^\dagger = \frac{1}{\sqrt{2e\hbar B}}(\pi_x + i\pi_y)$$

$$[a, a^\dagger] = 1 \qquad H = \frac{1}{2m}\boldsymbol{\pi}\cdot\boldsymbol{\pi} = \hbar\omega_B\left(a^\dagger a + \frac{1}{2}\right) \qquad E_n = \hbar\omega_B\left(n + \frac{1}{2}\right)$$

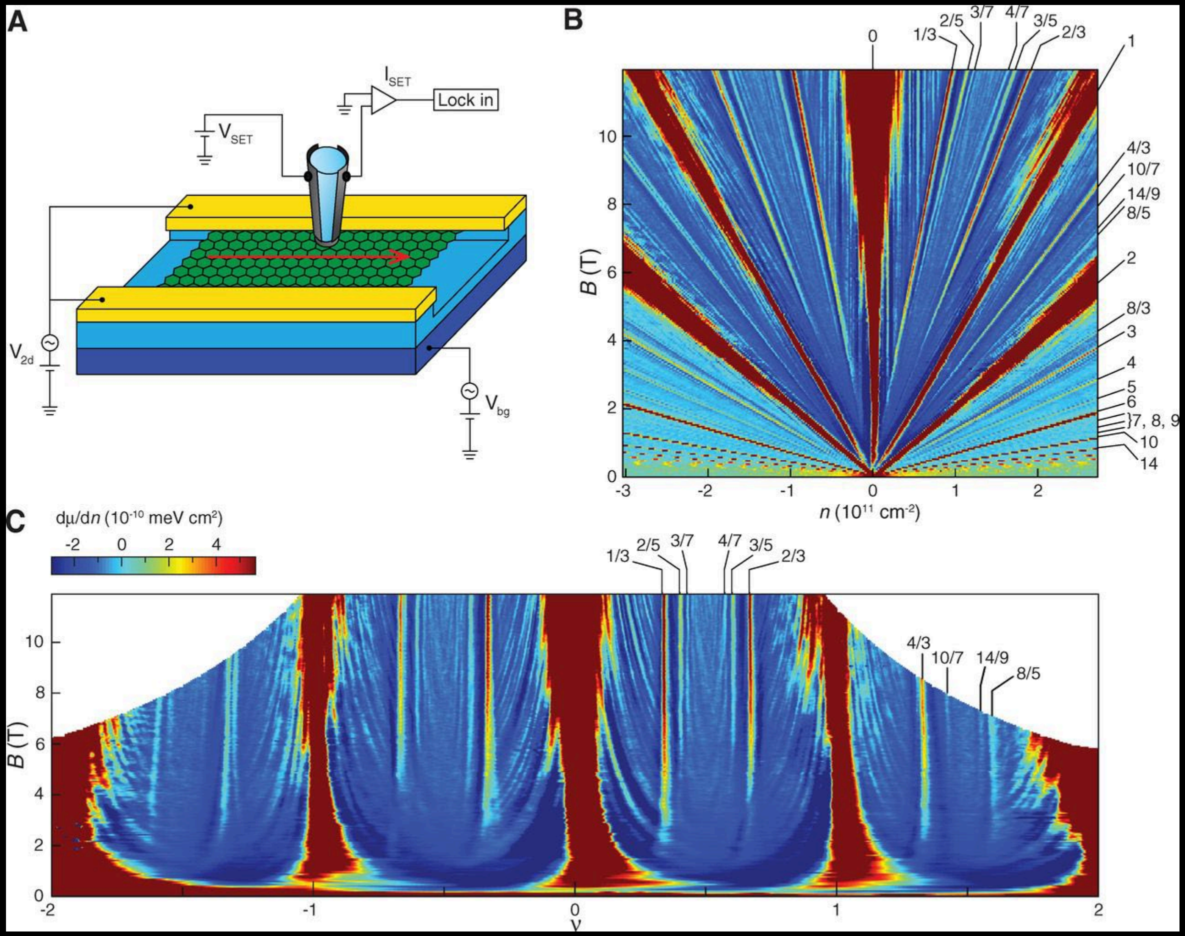
Electron transport

Quantum Hall effect



Kim et al. Nat. Commun. (2021)

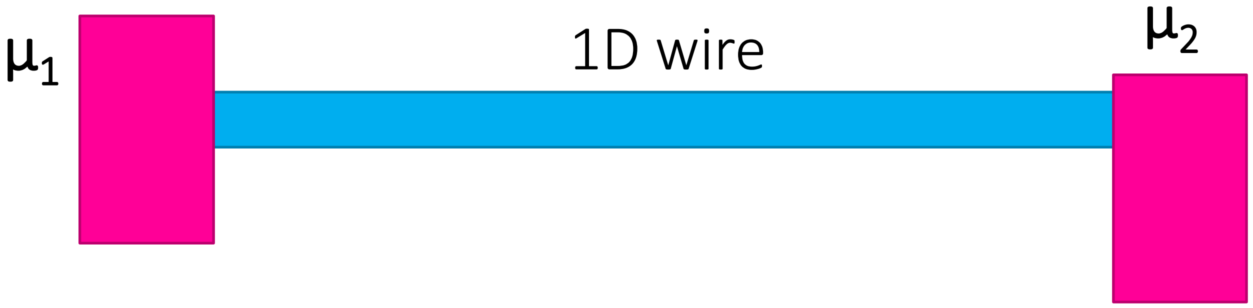
Fractional quantum Hall effect



Yacoby's group Science (2012)

Electron transport

Quantum wire(1D)



$$I = \int_{\mu_1}^{\mu_2} e\overset{\text{velocity}}{v}(E)\left(\underset{\text{spin}}{2}\overbrace{\underset{k>0}{\frac{1}{2}}g_{1D}(E)}^{\text{1D density}}\right)\mathrm{d}E = \int_{\mu_1}^{\mu_2} ev(E)\left(\frac{2}{hv(E)}\right)\mathrm{d}E$$

$$= \frac{2e}{h}(\mu_2 - \mu_1) = \frac{2e}{h}(eV)$$

$$\longrightarrow \quad G = I/V = \frac{2e^2}{h}$$

Conductance is a fixed value regardless of the length of the wire.

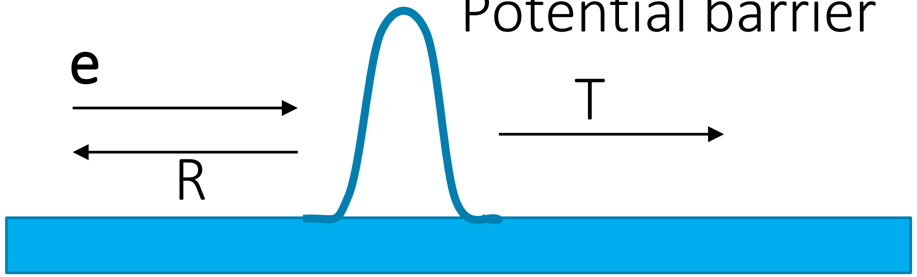
Jesper Nygård
Lecture note

Electron transport

Quantum wire(1D)

$$G = I/V = \frac{2e^2}{h}$$

Potential barrier



1D wire

Now become

Landauer formula

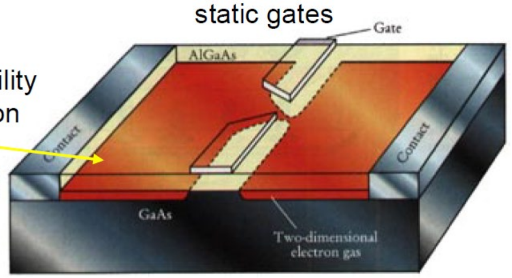
$$G = \frac{2e^2}{h}T \quad (\text{transmission probability } T)$$

With more channels (sub-bands)

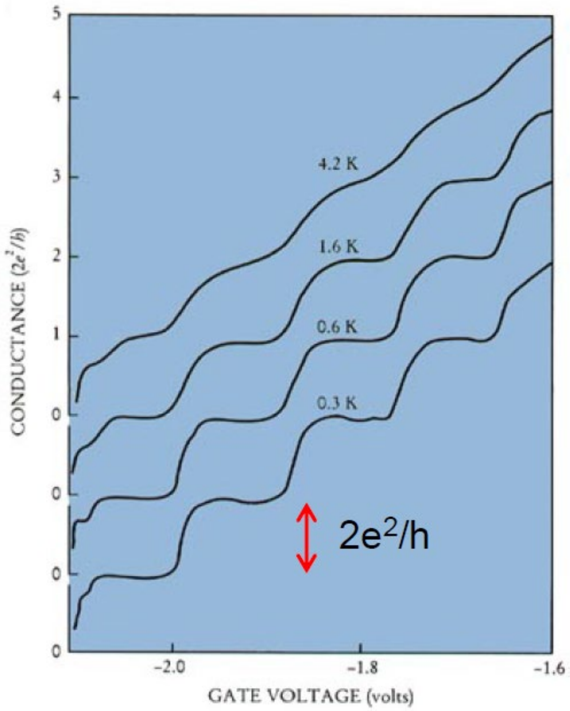
$$G(E_F) = \frac{2e^2}{h}\sum_n T_n(E_F) \ (T=0)$$

Depletion by electro-static gates

High mobility 2D electron gas ('ballistic')



'Quantum Point Contact'



Also good to review Landauer–Buttiker formalism

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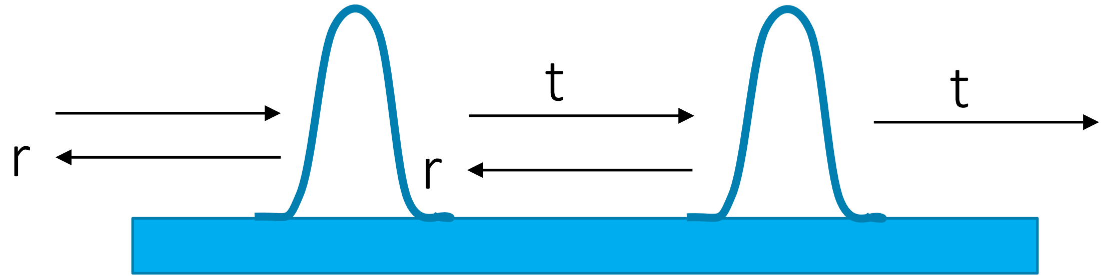
Lecture note

Electron transport

Quantum dot (0D system)

1D wire with two identical barriers under a **coherent transport**



$t = |t|e^{i\phi_t}$

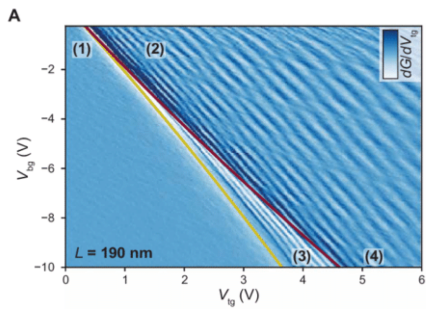
$r = |r|e^{i\phi_r}$

Total transmission:

$$T_{\text{total}} = \frac{|t|^4}{1 + |r|^4 - 2|r|^2\cos(\phi)}, \quad \phi = 2kL + \phi_{r1} + \phi_{r2}$$

Resonance condition:

$kL = \pi n$



For the resonant transmission(round trip with a phase of $2\pi n$**)**:

$$\text{For } \phi = 2\pi n: \quad T_{\text{total}} = \frac{|t|^4}{(1-|r|^2)^2} = \frac{T^2}{(1-R)^2} = \frac{T^2}{T^2} = 1$$

Resulting in a total transmission even $T=|t|^2<1$ for individual barriers

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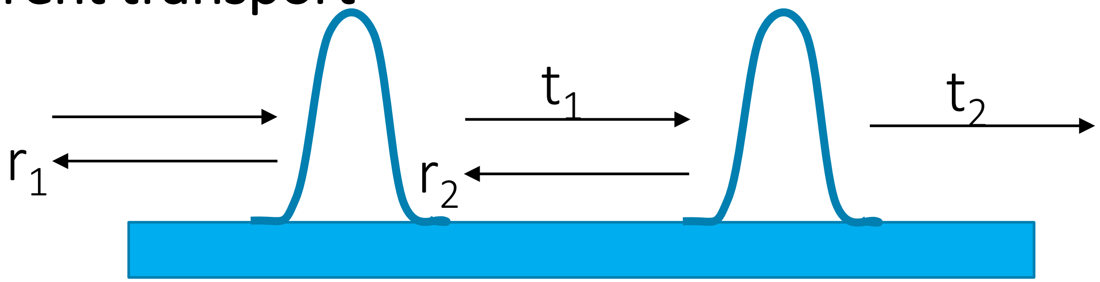
Lecture note

Electron transport

Quantum dot (0D system)

Incoherent transport



Total transmission:

$$T_{\text{total}} = \frac{|t_1|^2 |t_2|^2}{1 - |r_1|^2 |r_2|^2}$$

The resistance is like resistors in series:

$$\text{Resistance} = \frac{h}{2e^2}\left(1 + \frac{|r_1|^2}{|t_1|^2} + \frac{|r_2|^2}{|t_2|^2}\right)$$

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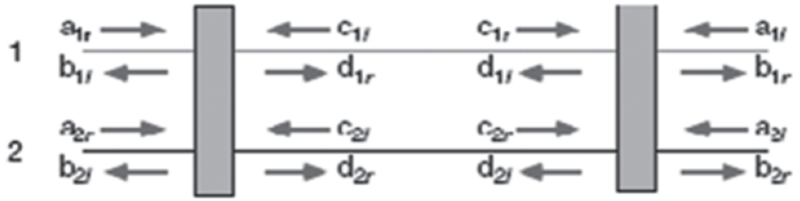
Lecture note

Scattering matrix:

$$\mathbf{b} = \tilde{S} \cdot \mathbf{a}. \qquad \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$



$$S_T = S_L \otimes S_N \otimes S_R$$

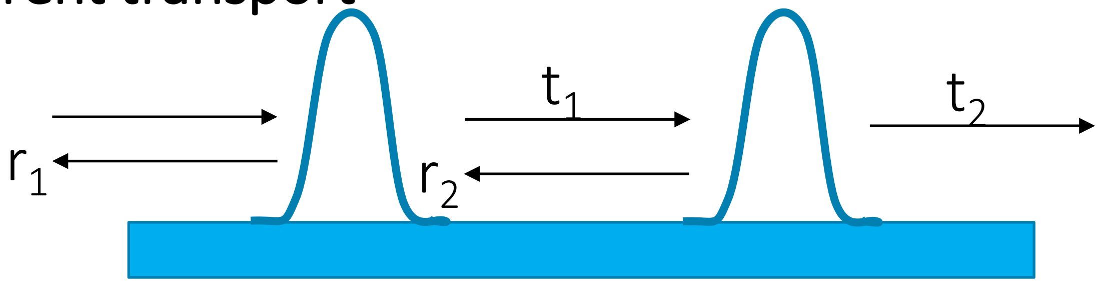
$$\begin{pmatrix} d_{1r} \\ d_{2r} \\ b_{1l} \\ b_{2l} \end{pmatrix} = S_L \begin{pmatrix} a_{1r} \\ a_{2r} \\ c_{1l} \\ c_{2l} \end{pmatrix} \qquad \begin{pmatrix} c_{1r} \\ c_{2r} \\ c_{1l} \\ c_{2l} \end{pmatrix} = S_N \begin{pmatrix} d_{1r} \\ d_{2r} \\ d_{1l} \\ d_{2l} \end{pmatrix} \qquad \begin{pmatrix} b_{1r} \\ b_{2r} \\ d_{1l} \\ d_{2l} \end{pmatrix} = S_R \begin{pmatrix} c_{1r} \\ c_{2r} \\ a_{1l} \\ a_{2l} \end{pmatrix}$$

$$\begin{pmatrix} b_{1r} \\ b_{2r} \\ b_{1l} \\ b_{2l} \end{pmatrix} = S_T \begin{pmatrix} a_{1r} \\ a_{2r} \\ a_{1l} \\ a_{2l} \end{pmatrix} = \begin{pmatrix} t_{1r,1r} & t_{1r,2r} & r_{1r,1l} & r_{1r,2l} \\ t_{2r,1r} & t_{2r,2r} & r_{2r,1l} & r_{2r,2l} \\ r_{1l,1r} & r_{1l,2r} & t_{1l,1l} & t_{1l,2l} \\ r_{2l,1r} & r_{2l,2r} & t_{2l,1l} & t_{2l,2l} \end{pmatrix} \begin{pmatrix} a_{1r} \\ a_{2r} \\ a_{1l} \\ a_{2l} \end{pmatrix}$$

Electron transport

Quantum dot (0D system)

Incoherent transport



Total transmission:

$$T_{\rm total} = \frac{|t_1|^2|t_2|^2}{1-|r_1|^2|r_2|^2}$$

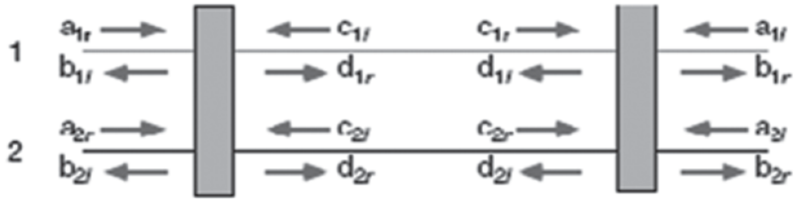
The resistance is like resistors in series:

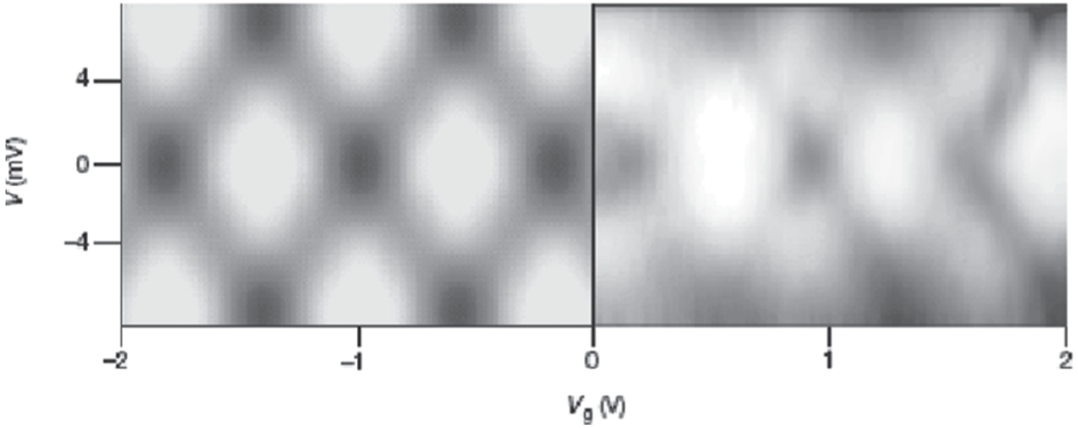
$$\text{Resistance} = \frac{h}{2e^2}\left(1+\frac{|r_1|^2}{|t_1|^2}+\frac{|r_2|^2}{|t_2|^2}\right)$$

Jesper Nygård
Lecture note

Scattering matrix:

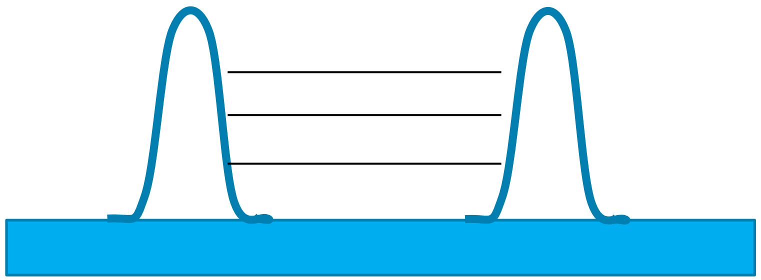
$$\mathbf{b} = \tilde{S}\cdot\mathbf{a}. \qquad \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

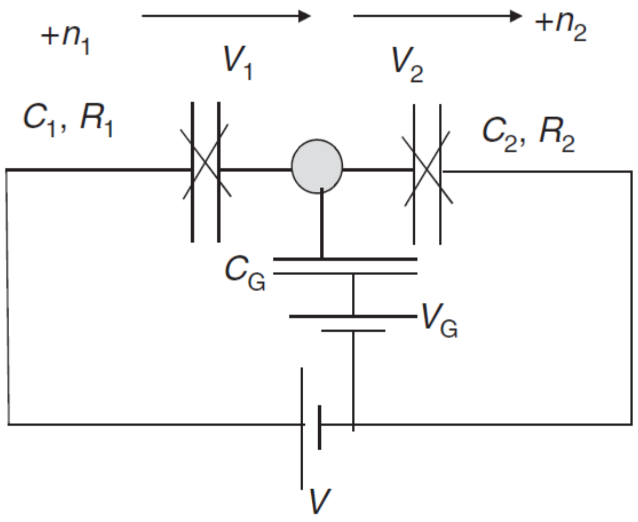




Electron transport

Quantum dot (single electron transition)





The total charge on the island

$$Q = Q_2 - Q_1 - Q_G \qquad \text{Where,} \quad Q_G = C_G(V_G - V_2)$$

The voltage drop on each junctions (barriers)

$$V_1 = \frac{1}{C_{tot}}\left((C_G + C_2)V_{DS} - C_G V_G + ne\right),$$

$$V_2 = \frac{1}{C_{tot}}\left(C_1 V_{DS} + C_G V_G - ne\right).$$

Total energy for adding e from left

$$\Delta E_{tot} = \frac{e}{C_{tot}}\left(\frac{e}{2} + (en + (C_2 + C_G)V_{DS} - C_G V_G)\right)$$

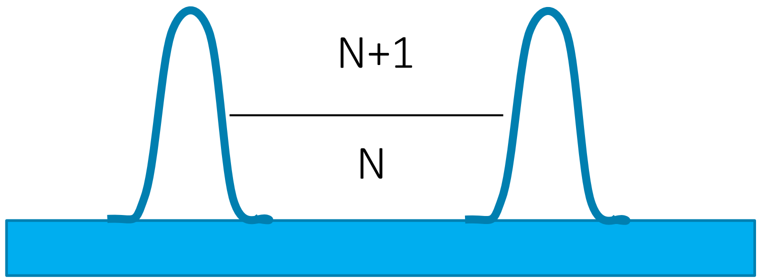
The conditions for tunneling one electron

$$\mp[en + (C_G + C_2)V_{DS} - C_G V_G] > \frac{e}{2} \quad \text{or}$$

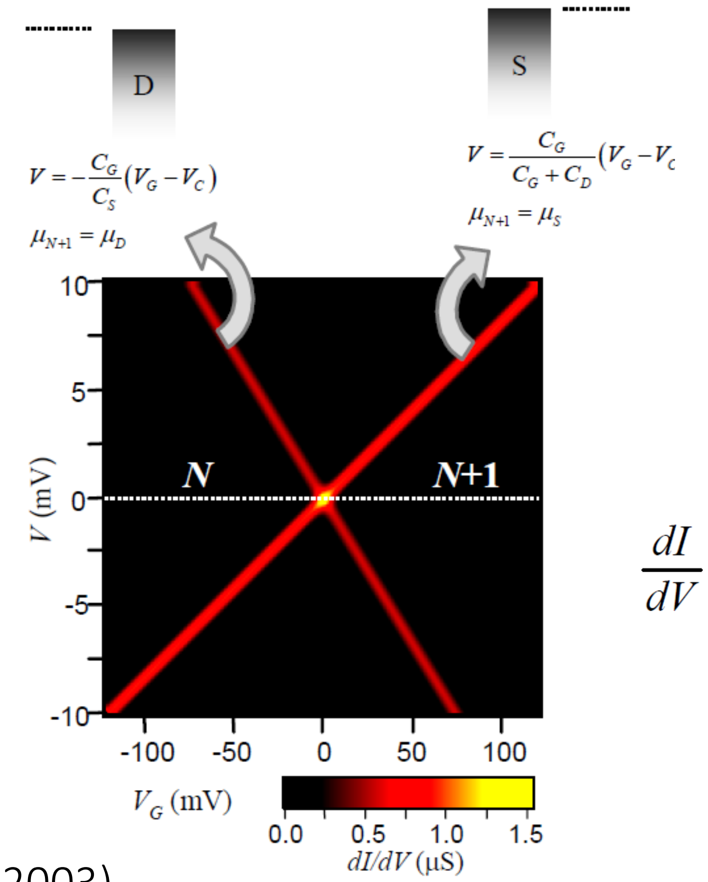
$$\pm[en - C_1 V_{DS} - C_G V_G] > \frac{e}{2}.$$

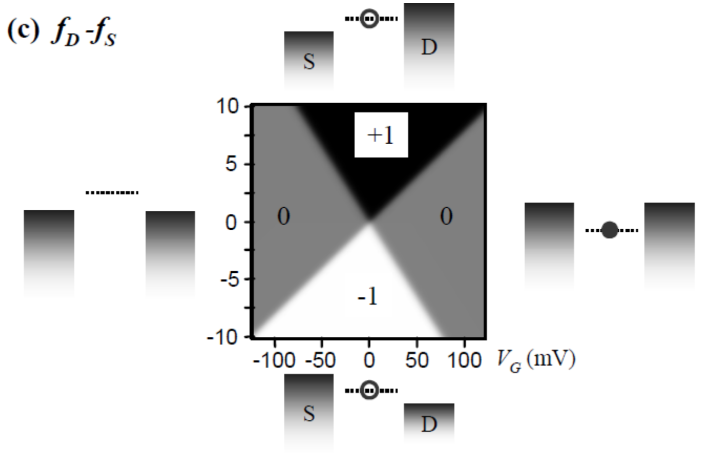
Electron transport

Quantum dot (single electron transition)



The conditions create the charge stability diagram. If we only consider a one-level system.





Jiwoong Park Ph.D. thesis (2003)

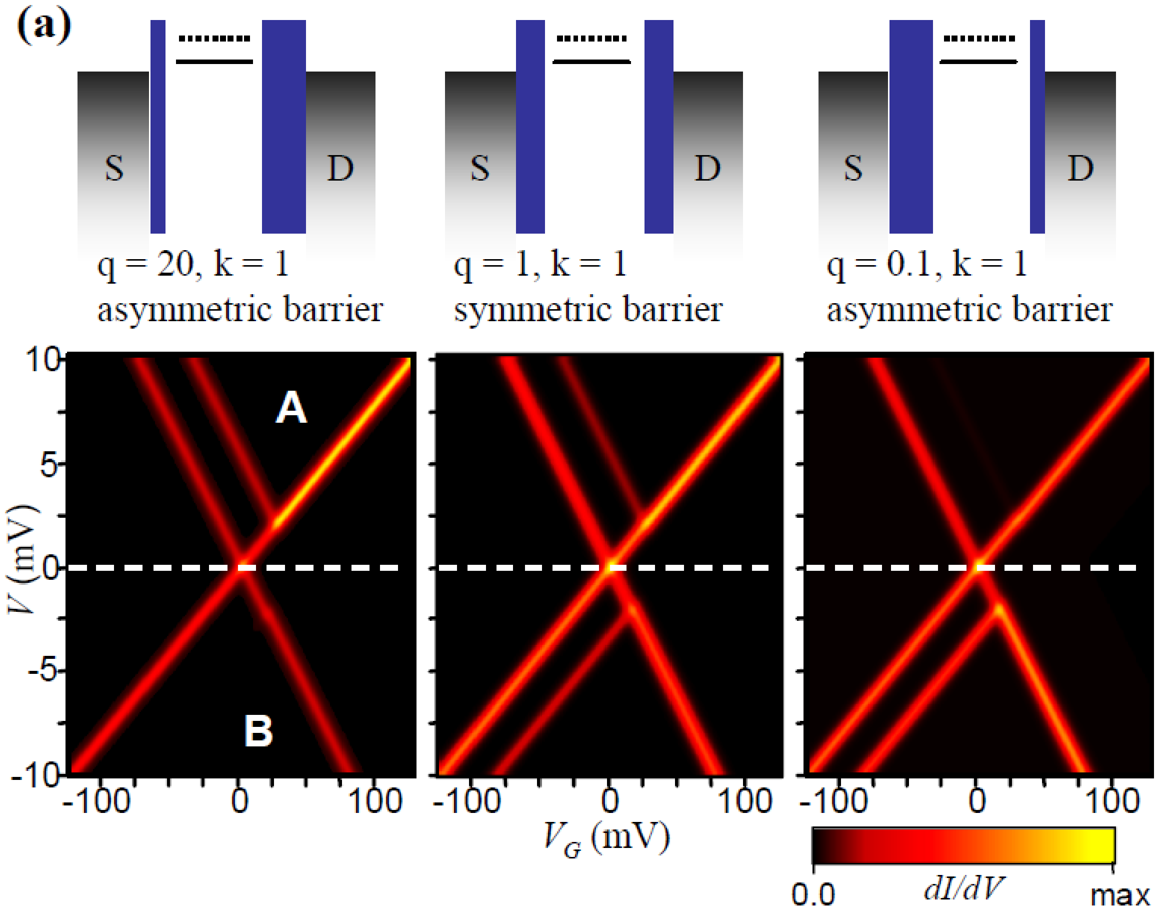
$$\frac{dI}{dV} = \frac{e^2\Gamma}{k_B T}\left((1 - f_D) f_D \frac{C_S}{C_{total}} + (1 - f_S) f_S \frac{C_D + C_G}{C_{total}} \right)$$

$$f_D = \left(1 + \exp\left(\frac{\mu_{N+1} - \mu_D}{k_B T}\right)\right)^{-1} = \left(1 + \exp\left(\frac{\mu_{N+1}}{k_B T}\right)\right)^{-1}$$

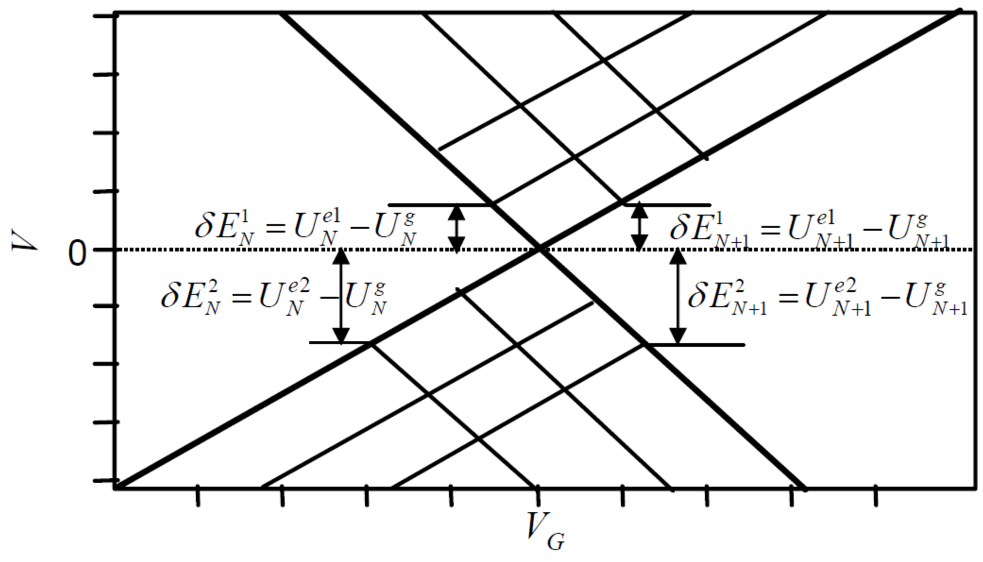
$$f_S = \left(1 + \exp\left(\frac{\mu_{N+1} - \mu_S}{k_B T}\right)\right)^{-1} = \left(1 + \exp\left(\frac{\mu_{N+1} + |e|V}{k_B T}\right)\right)^{-1}$$

Electron transport

Quantum dot (single electron transition)



Multi-levels quantum dot

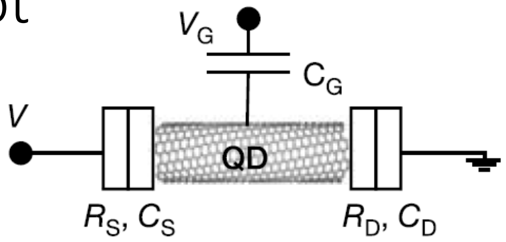


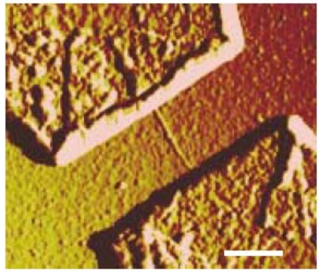
Jiwoong Park Ph.D. thesis (2003)

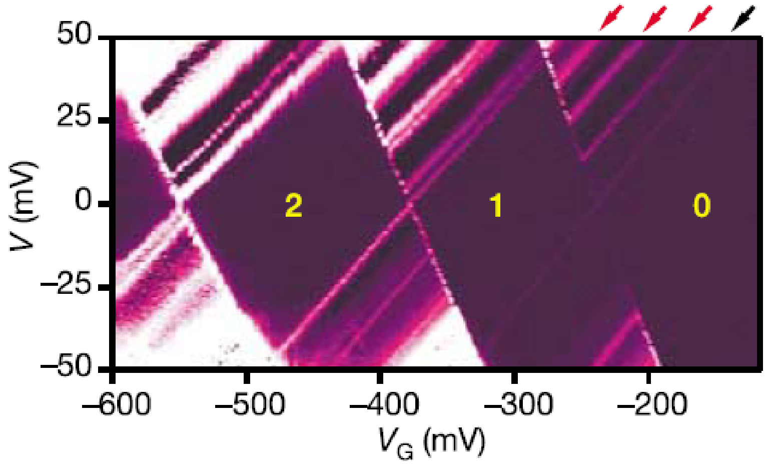
Electron transport

Quantum dot (single electron transition)

Carbon nanotube quantum dot

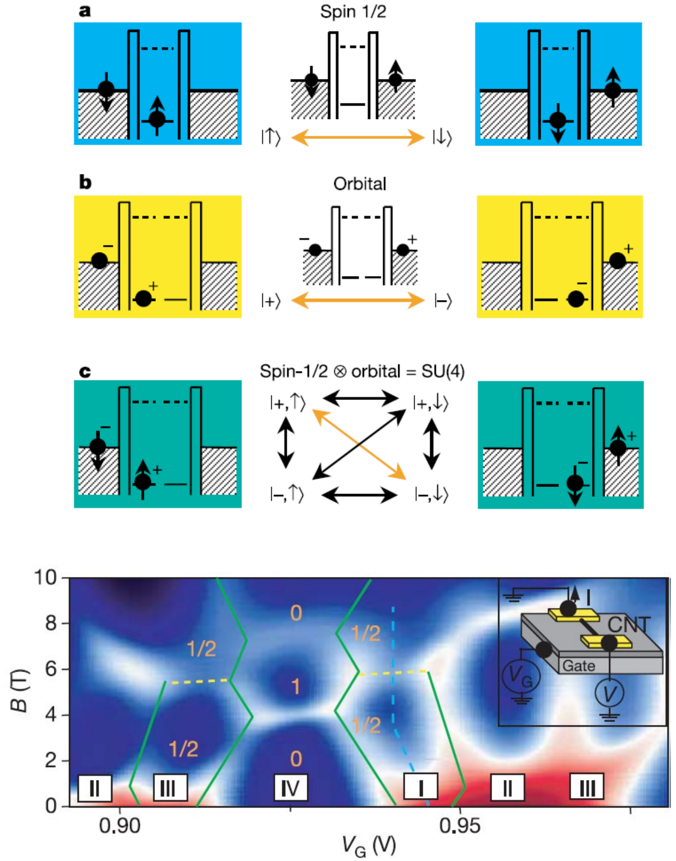






P. Jarillo-Herrero et al. Nature (2004)

Spin related cotunneling process: Kondo effect

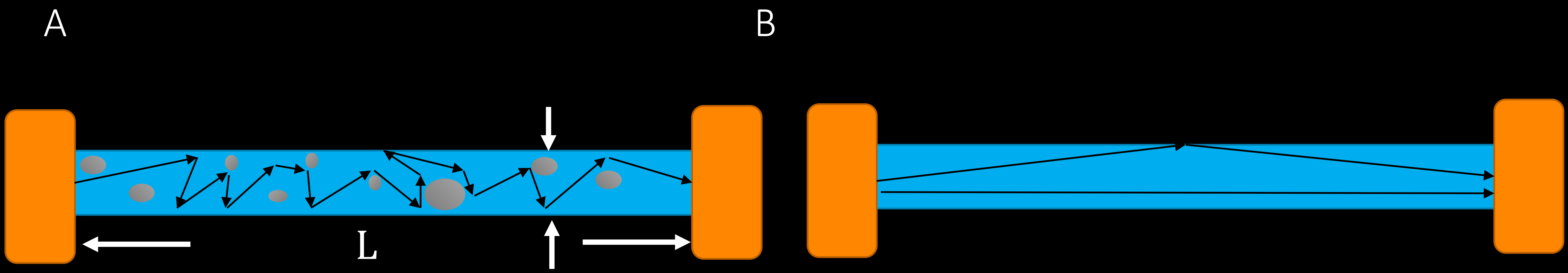


P. Jarillo-Herrero et al. Nature (2005)

Quizs

1. What is the mean free path?

2. which is diffusive and which is ballistic



Nanofabrication

How to manipulate materials

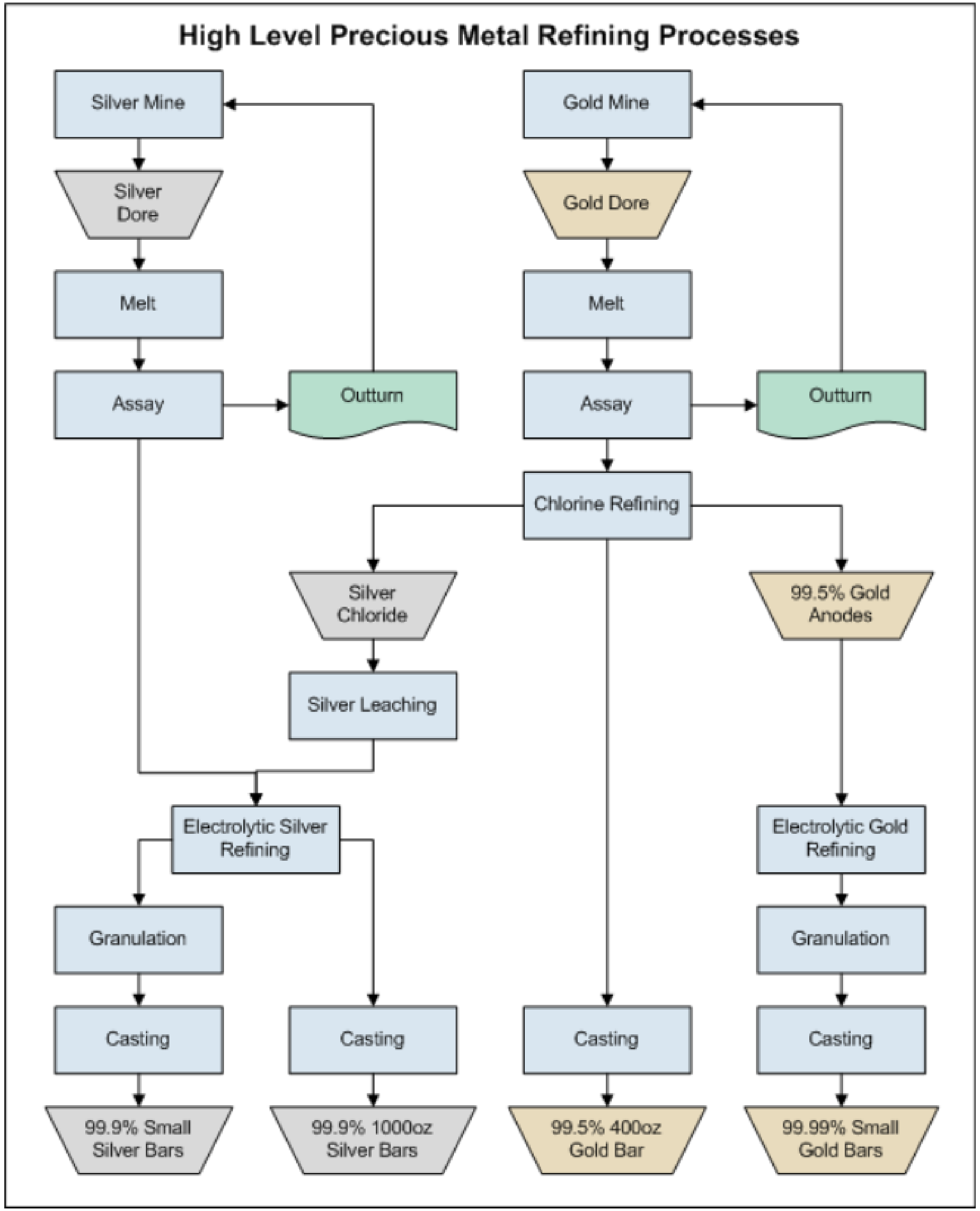
Nanofabrication

How to make materials?

To be honest, it is very complicated!!

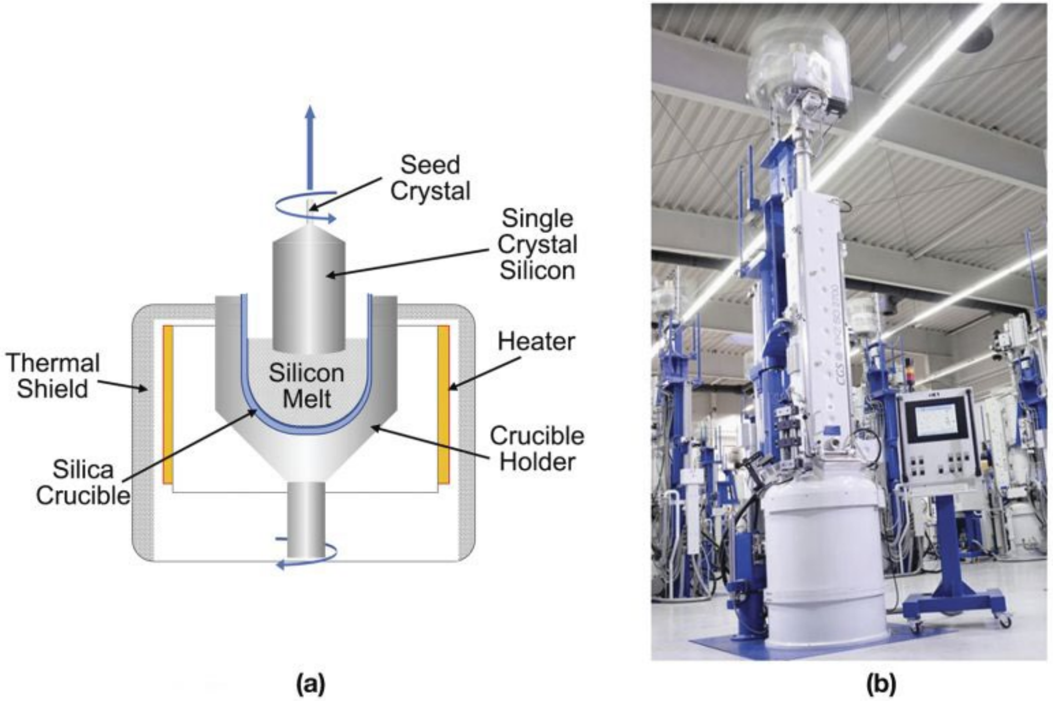
These are examples of two materials (Au and Ag) coming from mining. For other materials, there are a lot of different processes for various metals.

Adapted from Quora

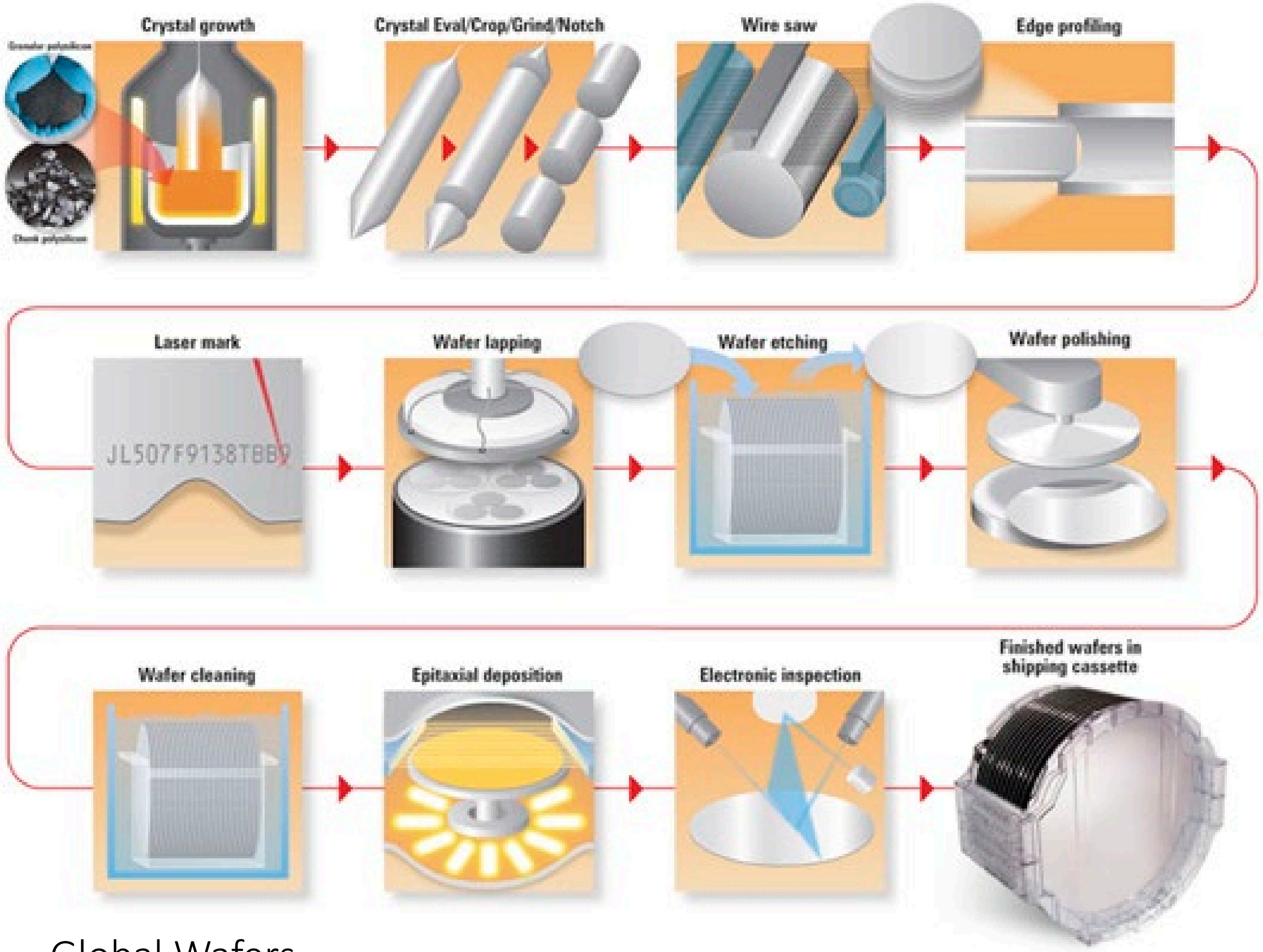


Nanofabrication

For semiconductors, Si is the most common one.



https://www.mksinst.com/n/silicon-wafer-production
MKS, (PVA TePla AG 2017)

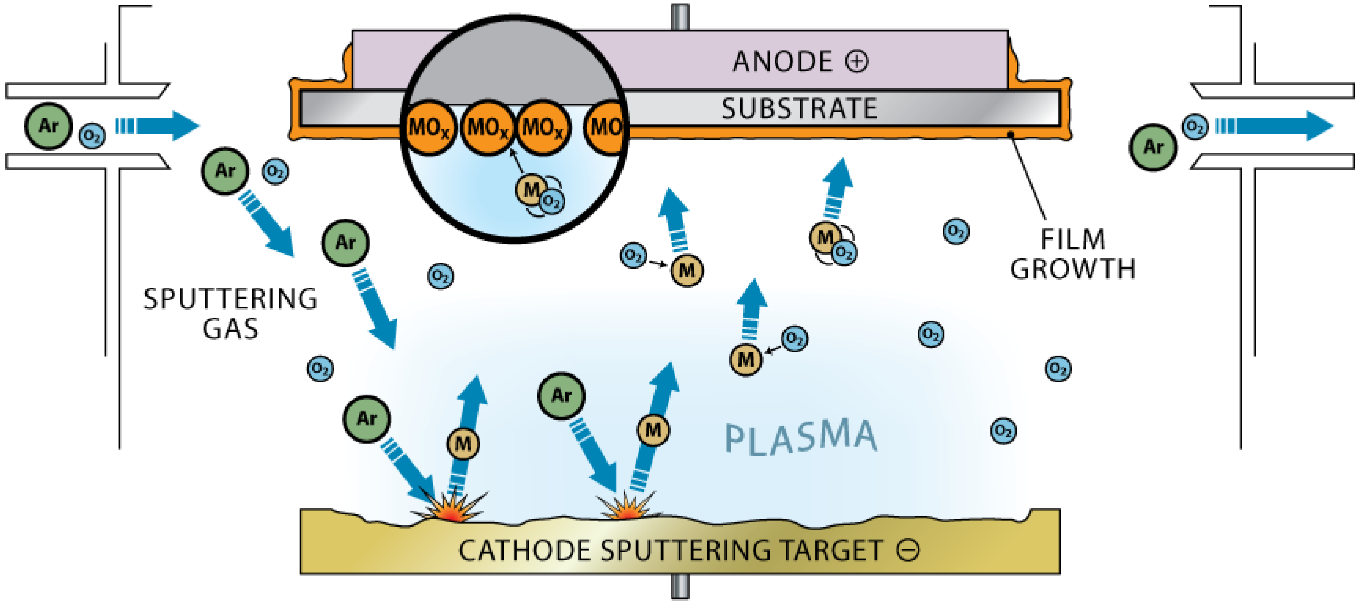


Global Wafers

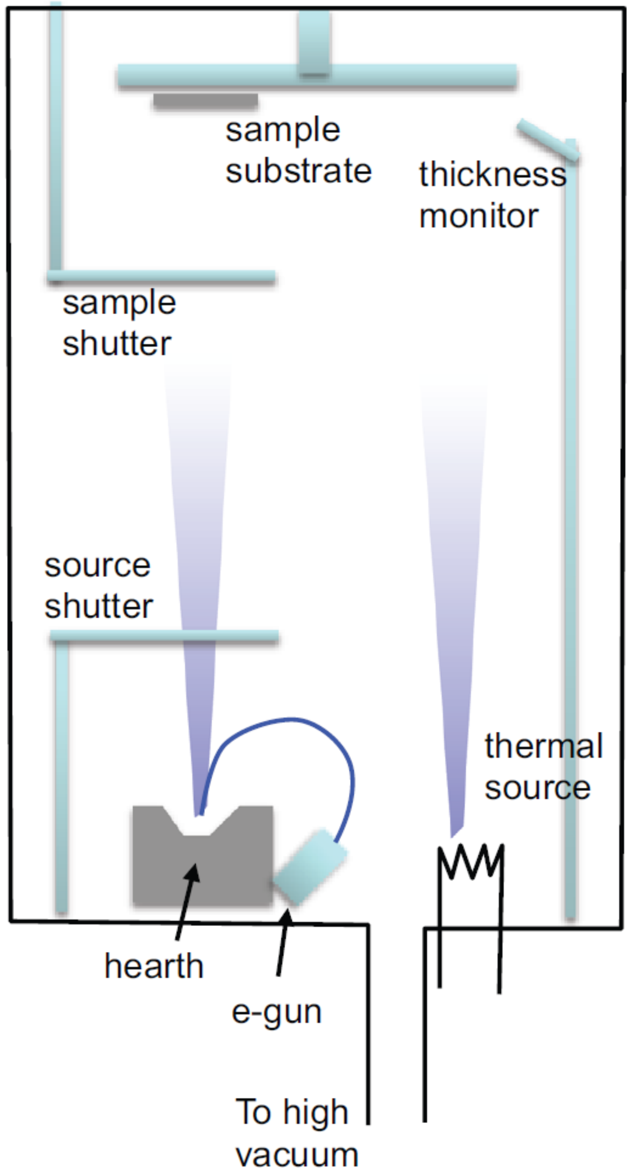
Nanofabrication

Physical Vapor Deposition

Sputtering



Evaporation

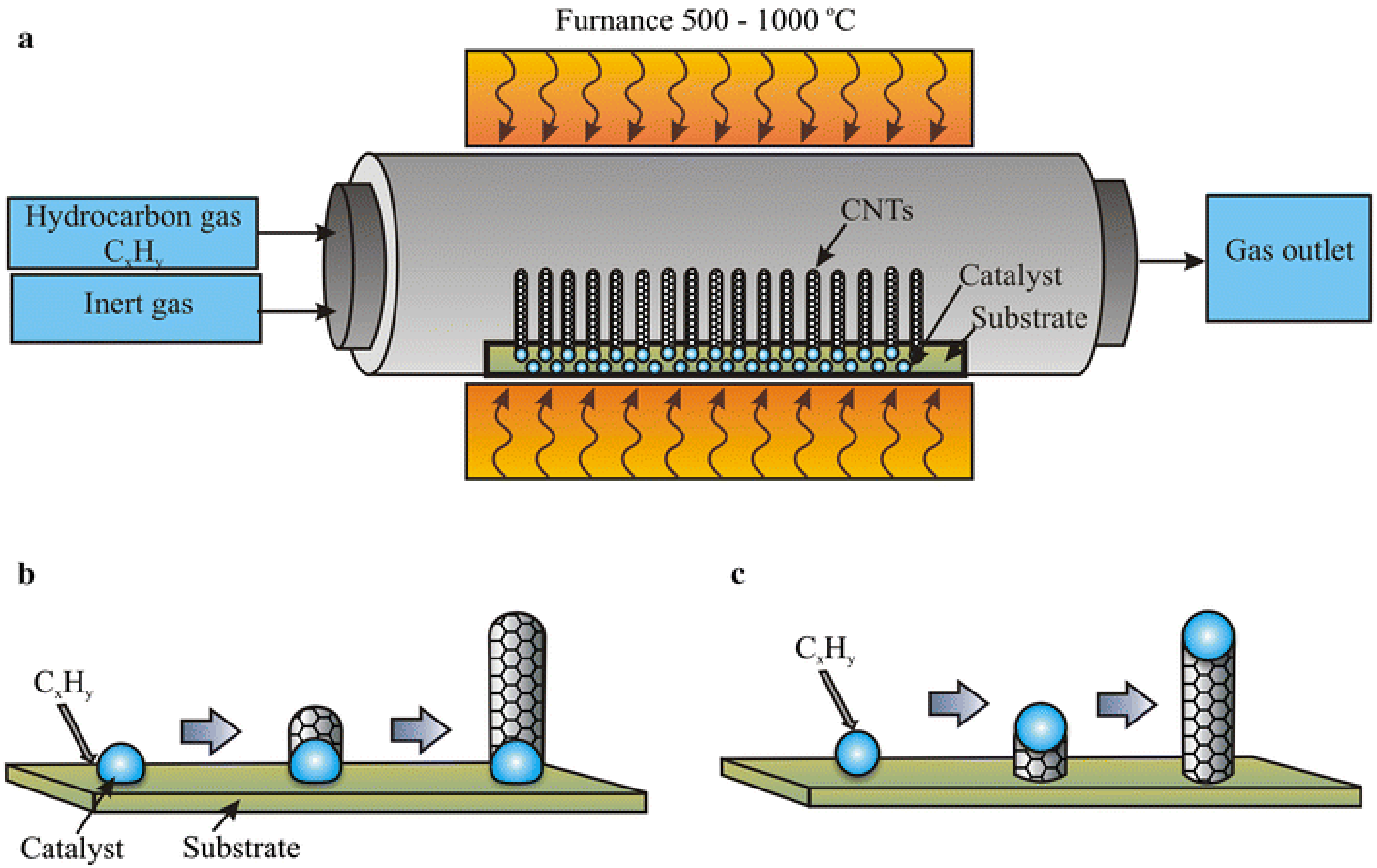


Nanofabrication

Chemical Vapor Deposition

Unlike the previous PVD, here we utilize the chemical reaction of gaseous chemicals (precursor compounds) at different temperatures for which chemicals may decompose then react in situ.

The sample can be grown on a substrate with a catalyst as the nucleation center. Often consider as an easy but dirty method.

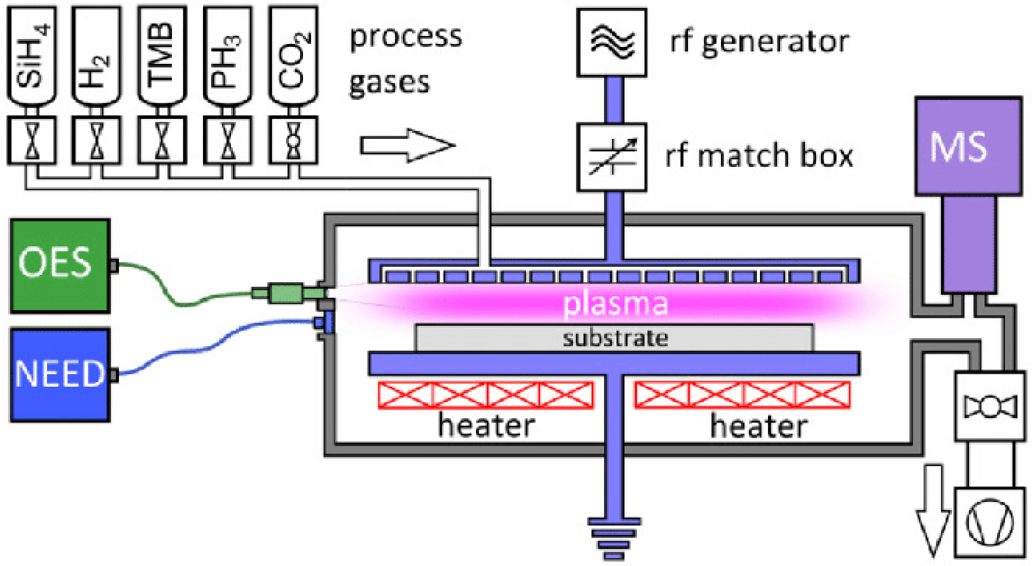


Olga Zaytseva et al. DOI:10.1186/s40538-016-0070-8

Nanofabrication

Plasma-Enhanced Chemical Vapor Deposition (PECVD)

Plasma reduces the required temperature for precursors, less power consumption.



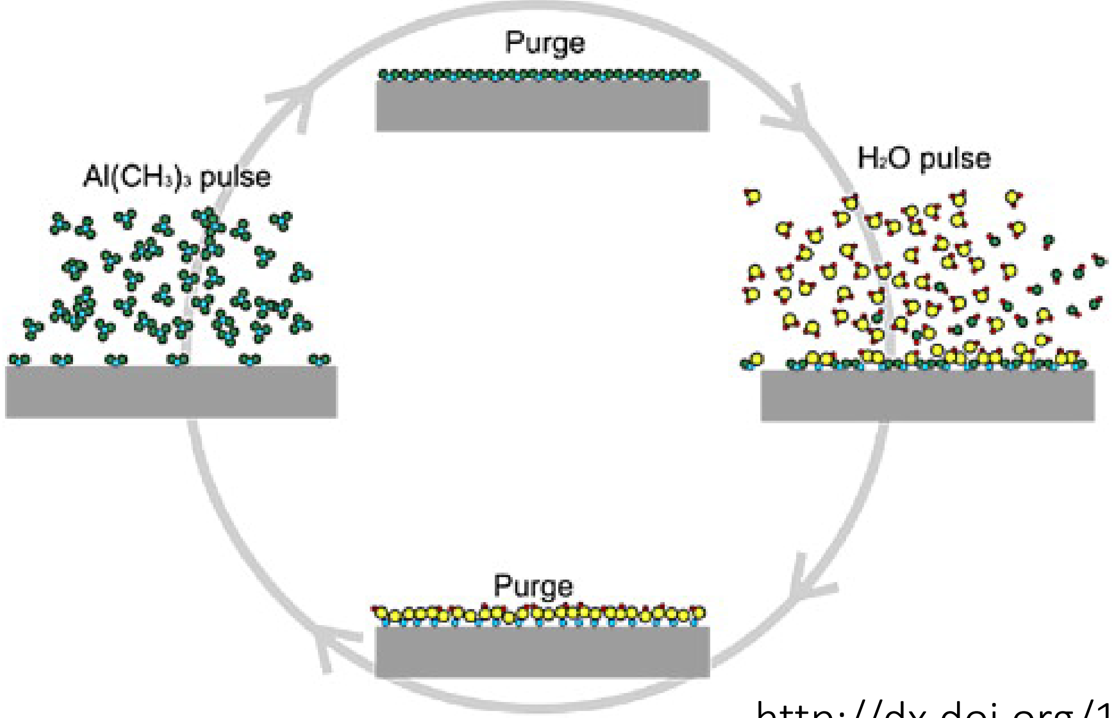
Onno Gabriel et al. EPJ Photovoltaics (2014)

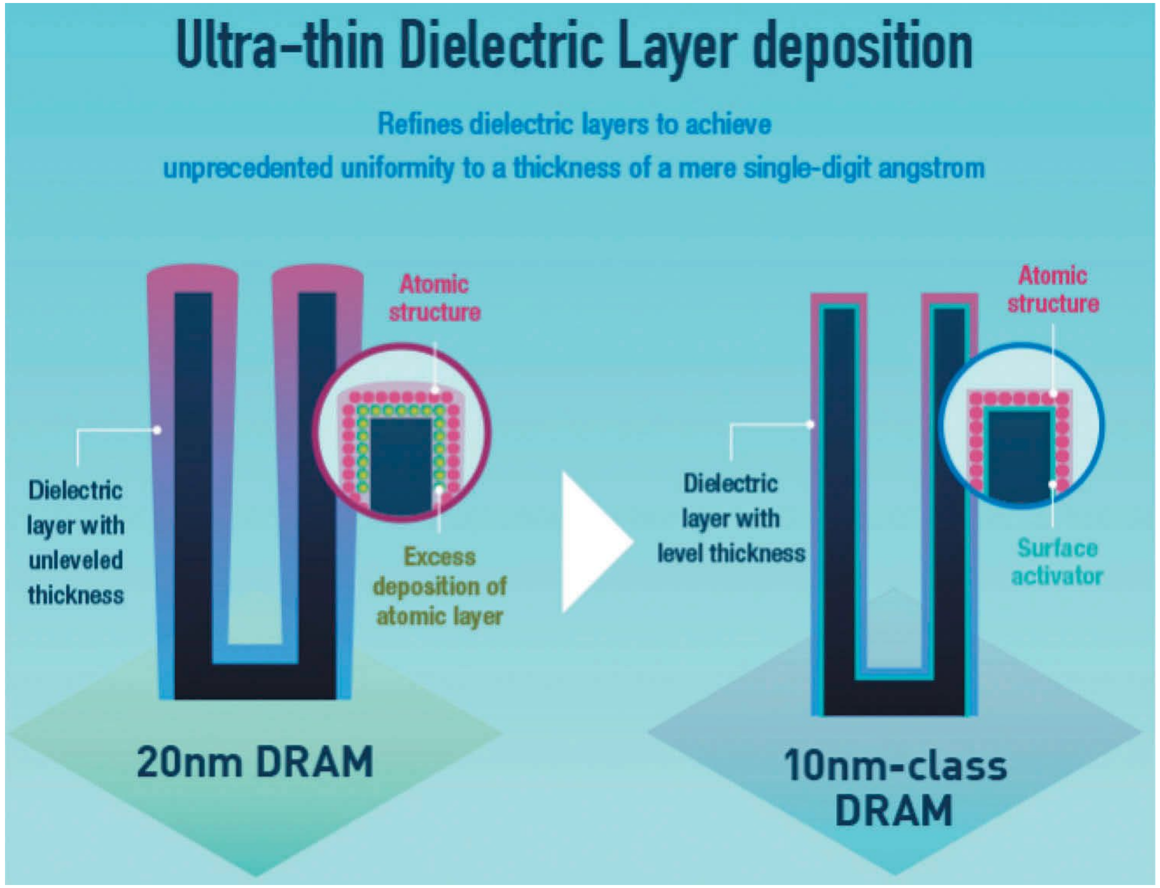
Applied Materials

Nanofabrication

Atomic Layer Deposition (ALD)

- A layer by layer growth process
- Very slow but good coverage



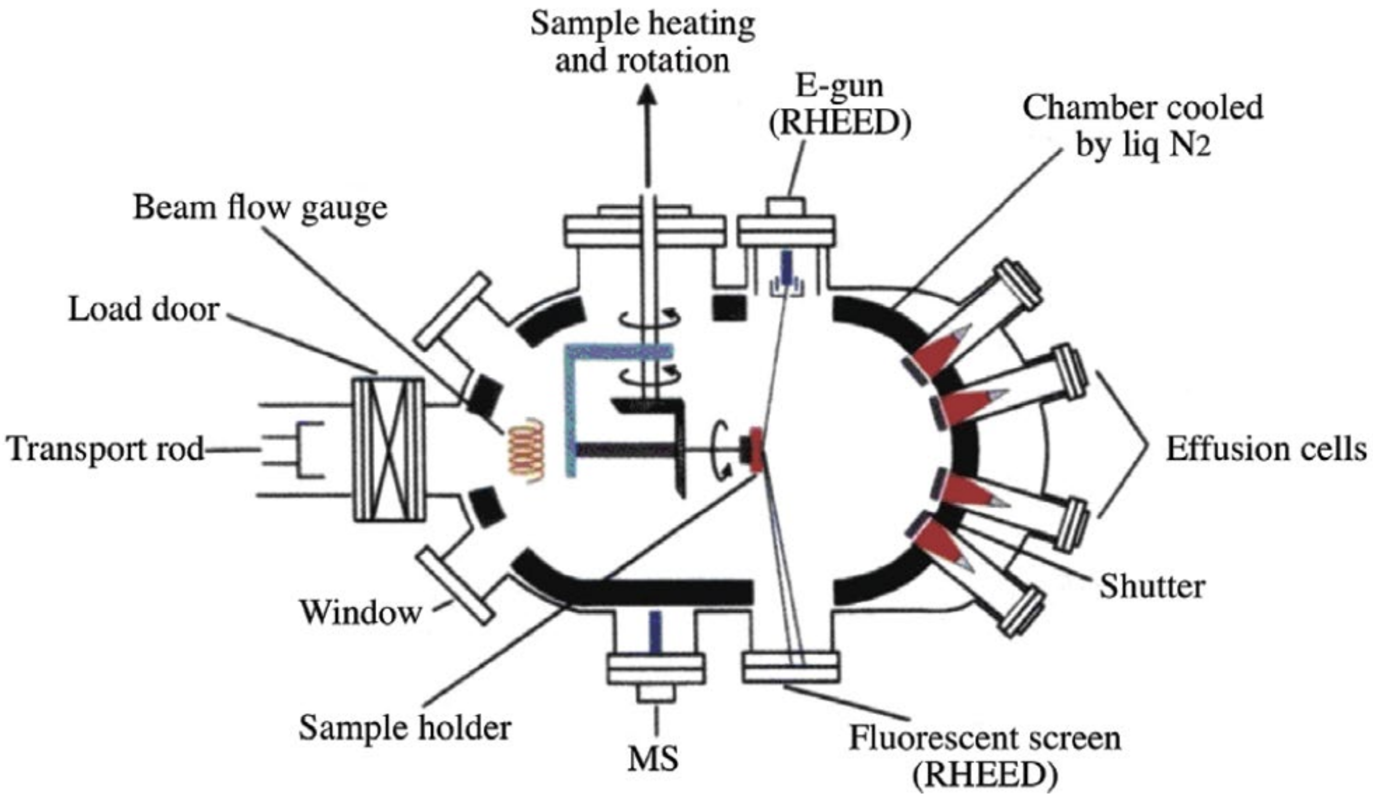


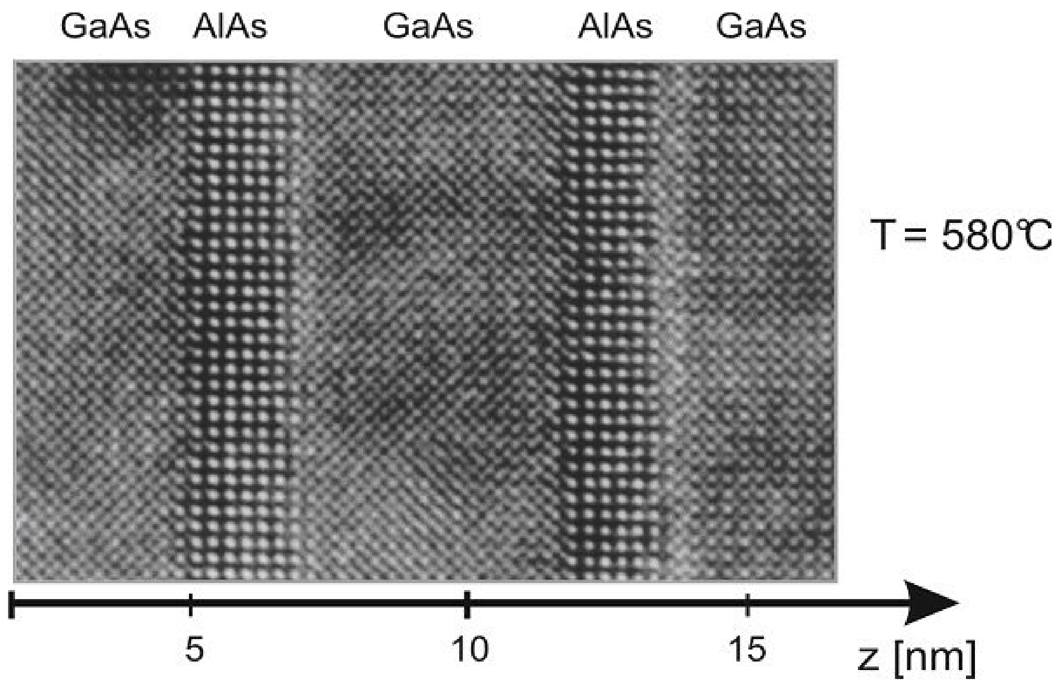
Samsung DRAM

http://dx.doi.org/10.1117/2.1201204.004218

Nanofabrication

Molecular beam epitaxy (MBE)





Simone Montanari Ph.D. thesis (2005)

- Atomically layer by layer growth process
- Very slow but extremely high quality
- UHV system, very expensive

Nanofabrication

Material Removing techniques

One can classify them into:

Chemical remove

Physical remove

Physical Chemical remove

Nanofabrication

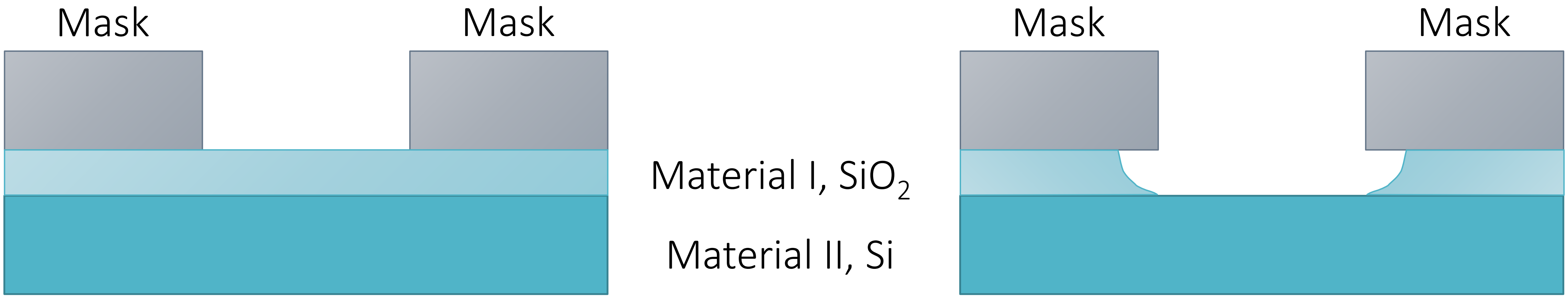
Chemical remove

A wet process, one can use chemicals to remove the unwanted materials.

Take SiO2 as an example:

$$SiO_2 + 6\ HF \rightarrow H_2SiF_6 + 2\ H_2O$$

Solvents: HF, BOE NH_4F

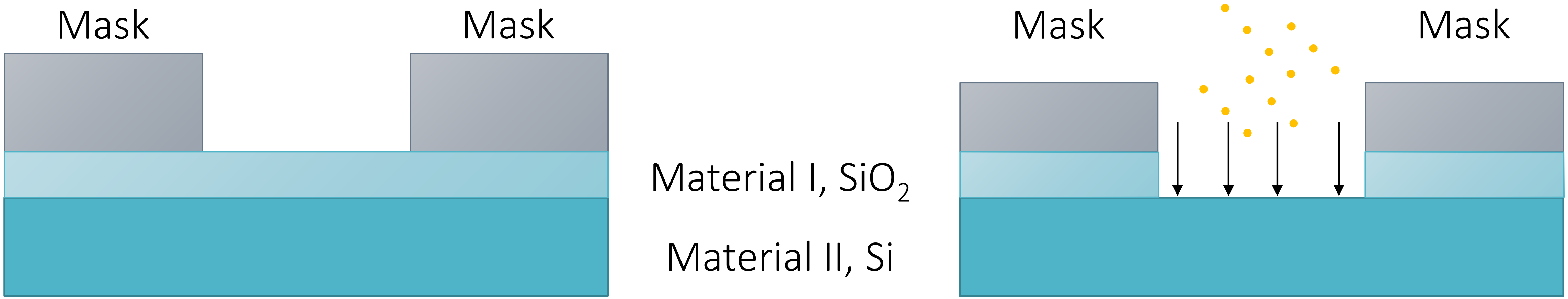


Etching rate(BOE_7:1): to SiO2 80 nm/min at 20 C, to Si is almost zero, how about mask??

Nanofabrication

Physical remove

A dry etching process, use gas particle in a plasma environment to form ion bombardment on the material surface.



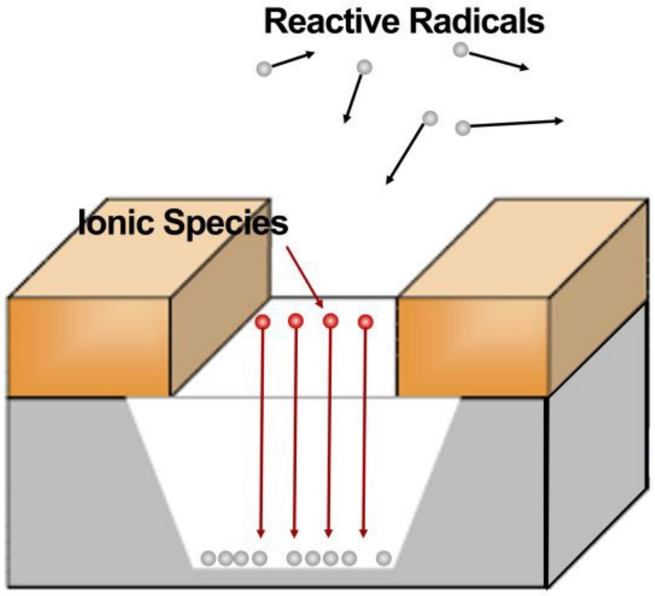
Etching rate: the same for everyone, therefore carefully calibration or endpoint detection is important

Nanofabrication

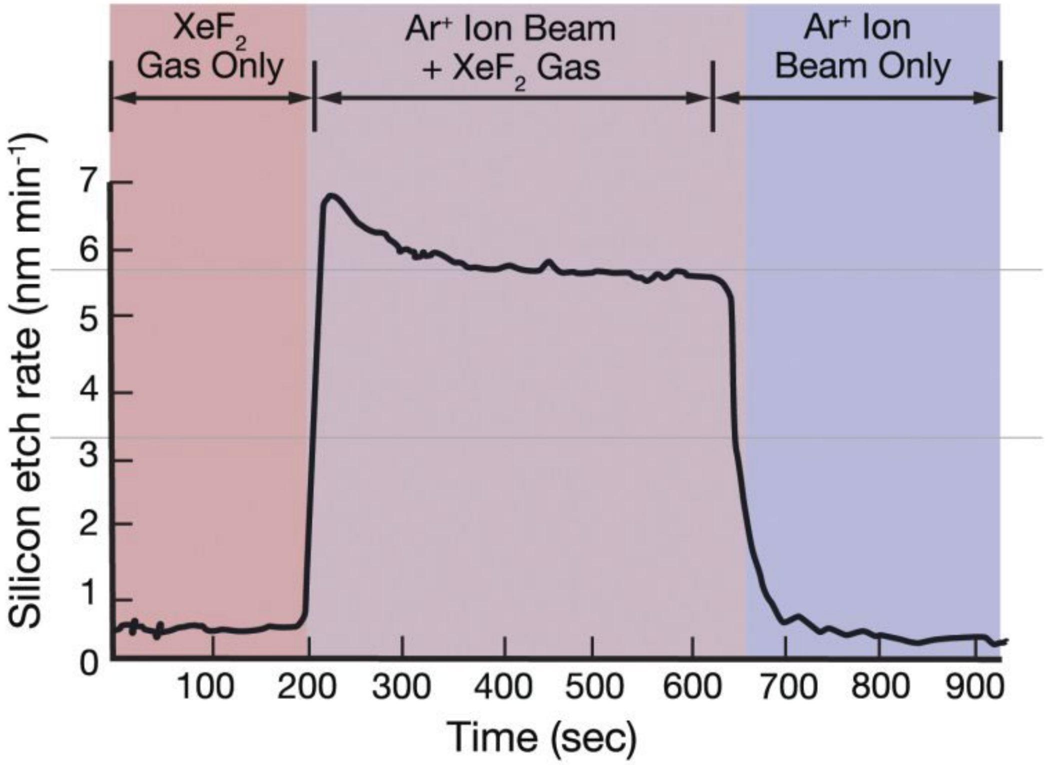
Chemical and Physical remove

A combined process uses ions to enhance the chemical reaction.

<u>Reactive Ion Etching</u>



Chemical etch is enhanced by ion bombardment



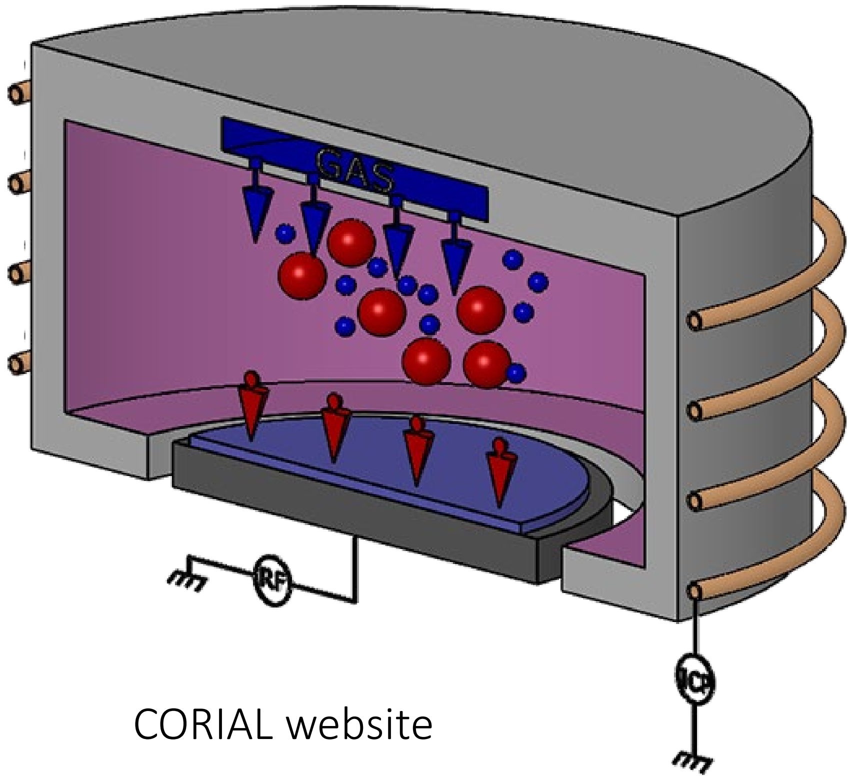
MKS website

Nanofabrication

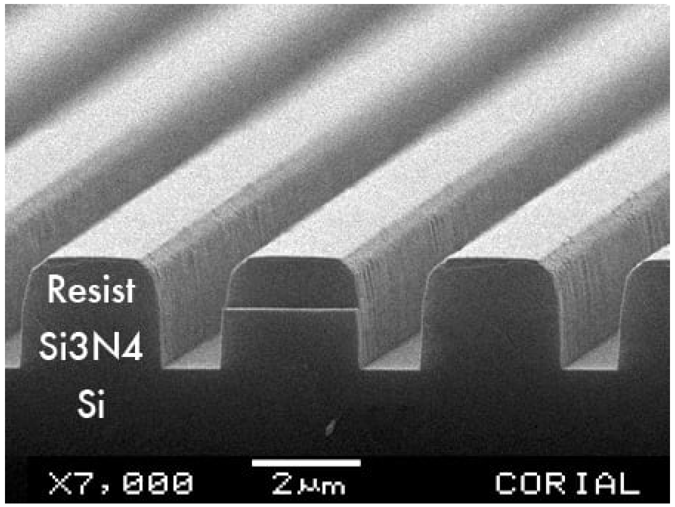
For high aspect ratio or VIA hole, Bosch process is needed

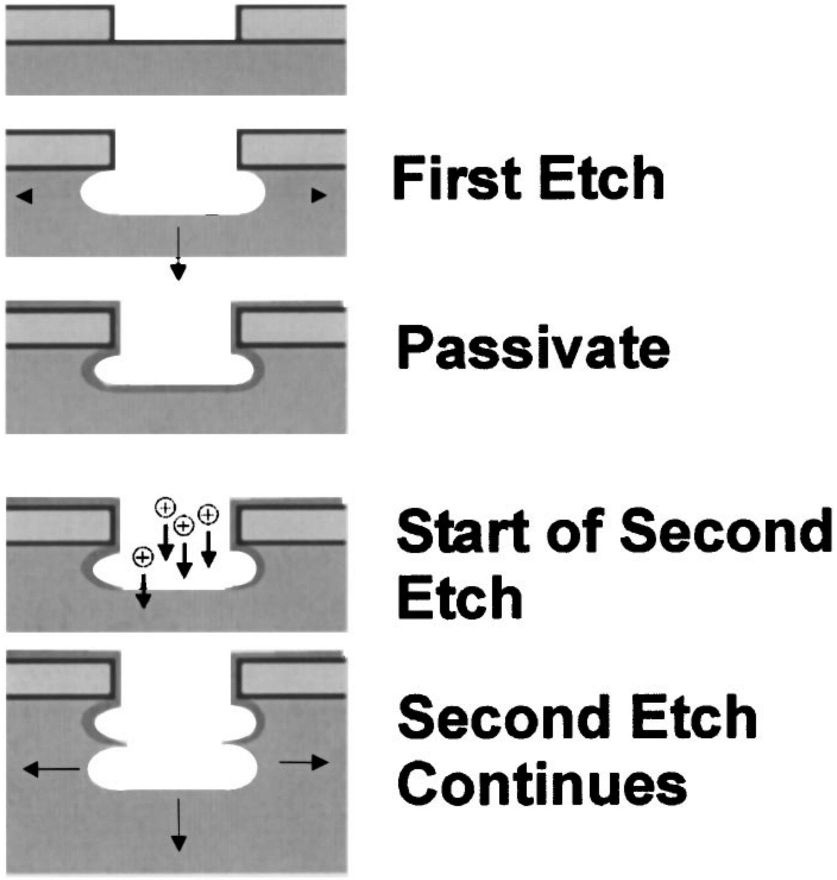
Chemical and Physical remove

<u>Inductively Coupled Plasma-RIE etch</u>



CORIAL website



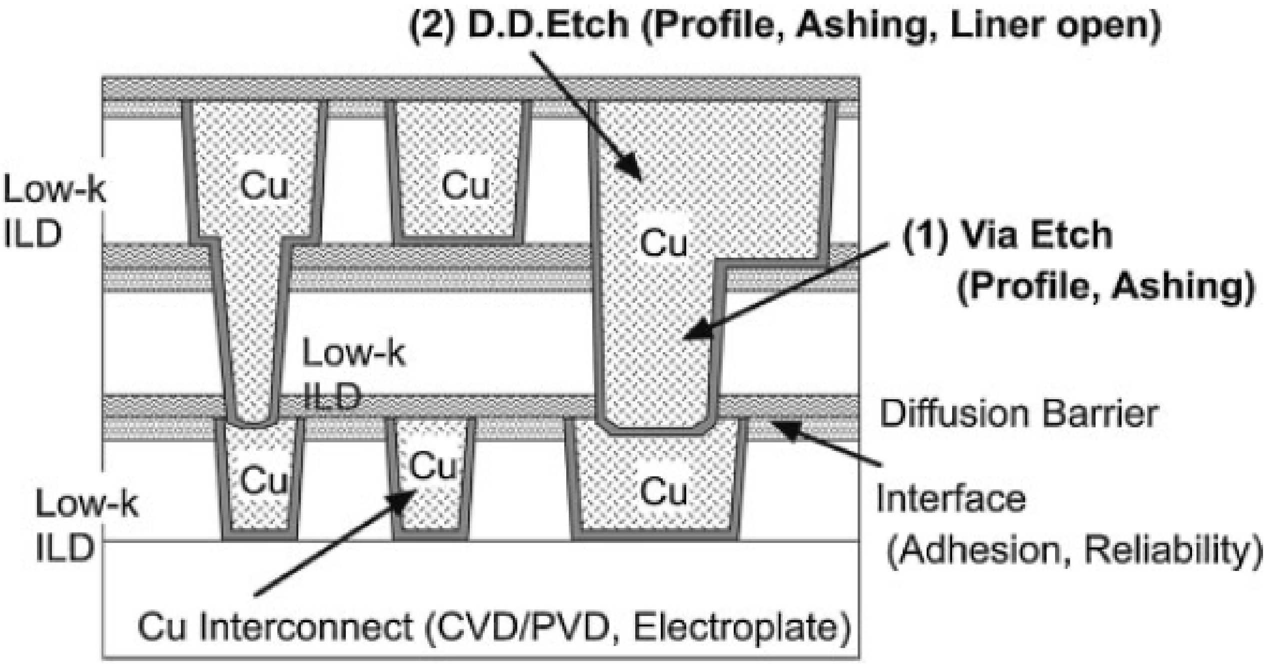


C. J. D. Craigie et al
http://dx.doi.org/10.1116/1.1515910

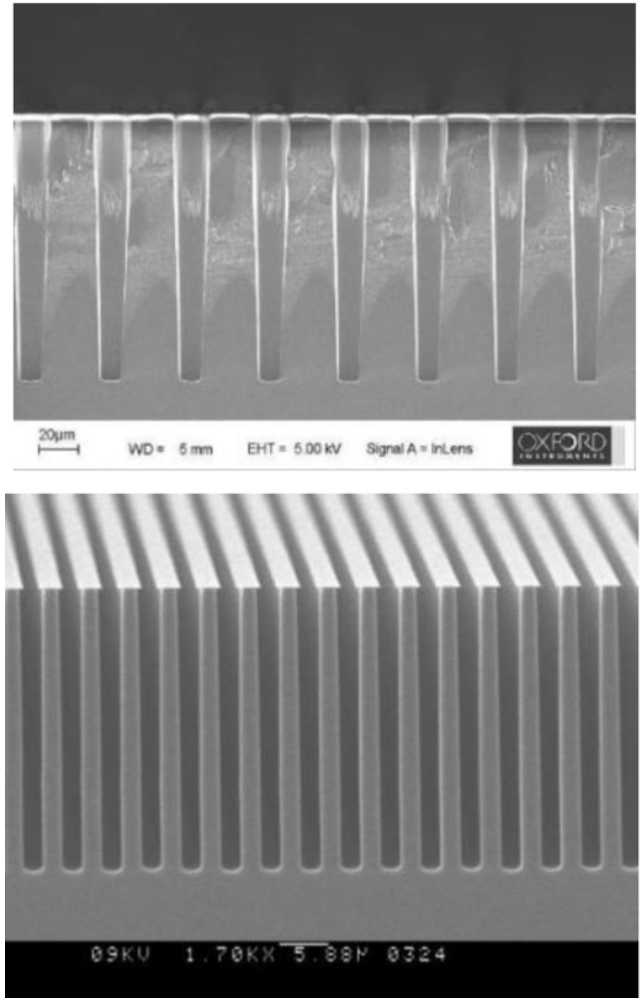
Nanofabrication

Chemical and Physical remove

Advanced etching technique is important!!!



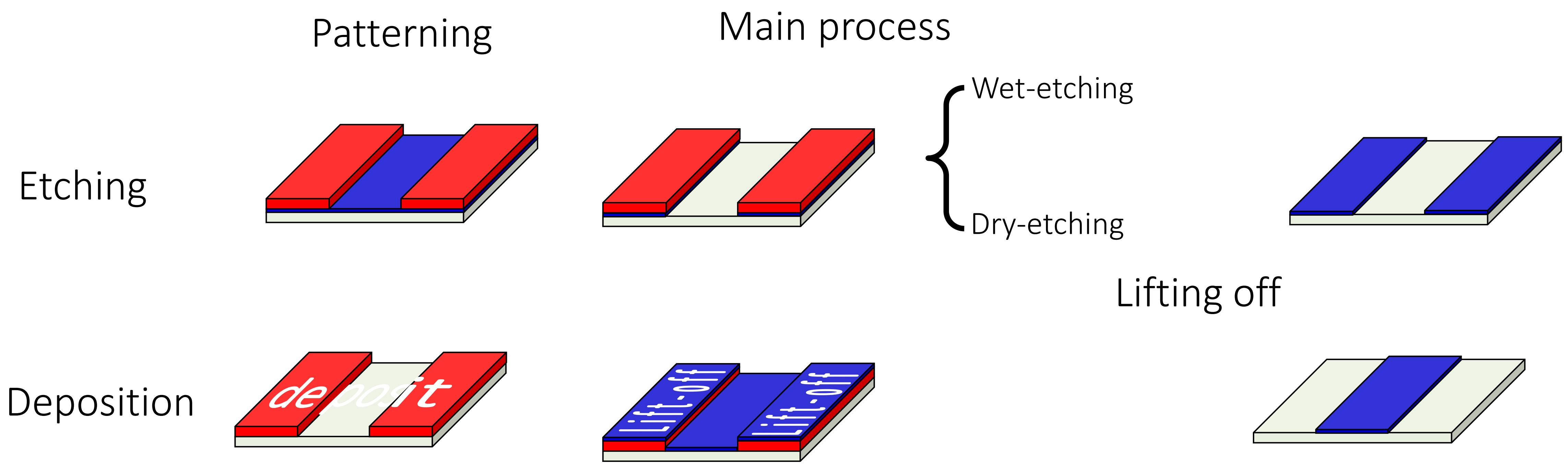
Haruhiko ABE et al. JJAP (2008)



AZoNano.com/Oxford

Nanofabrication

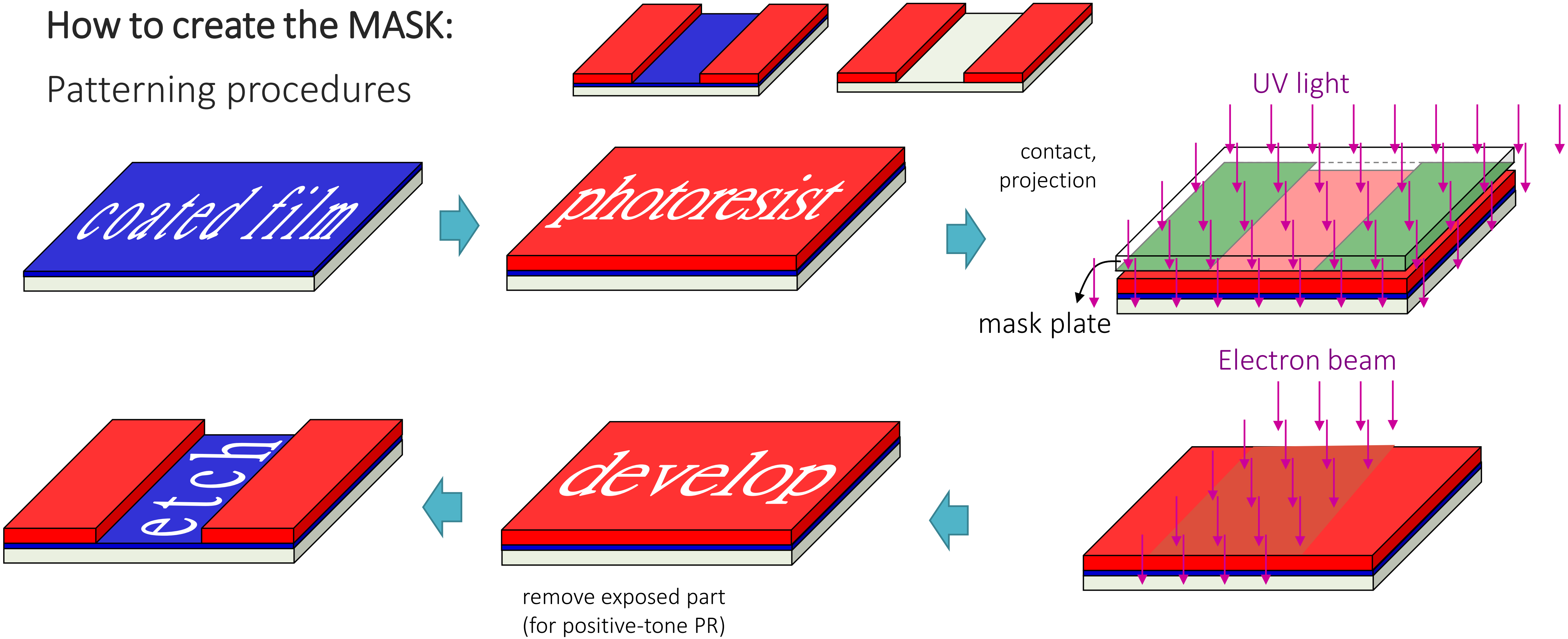
How to create the MASK: the workflow for lithography



Nanofabrication

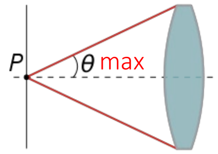
How to create the MASK:

Patterning procedures



Nanofabrication

Resolution: $R = k_1 (\lambda / NA)$



Depth of Focus $DoF = k_2 (\lambda / NA^2)$

NA = sinθ = numerical aperture

K_1 = a constant for a specific lithography process
smaller K1 can be achieved by
improving the process or resist contrast

Calculated R and DoF values

UV wavelength	248 nm	13.4 nm
Typical NA	**0.75**	**0.25**
Production value of k_1	**0.5**	**0.5**
Resolution	**0.17 µm**	**0.027 µm**
DoF (assuming k_2 = 1)	**0.44 µm**	**0.21 µm**

SOURCES OF RADIATION FOR MICROLITHOGRAPHY

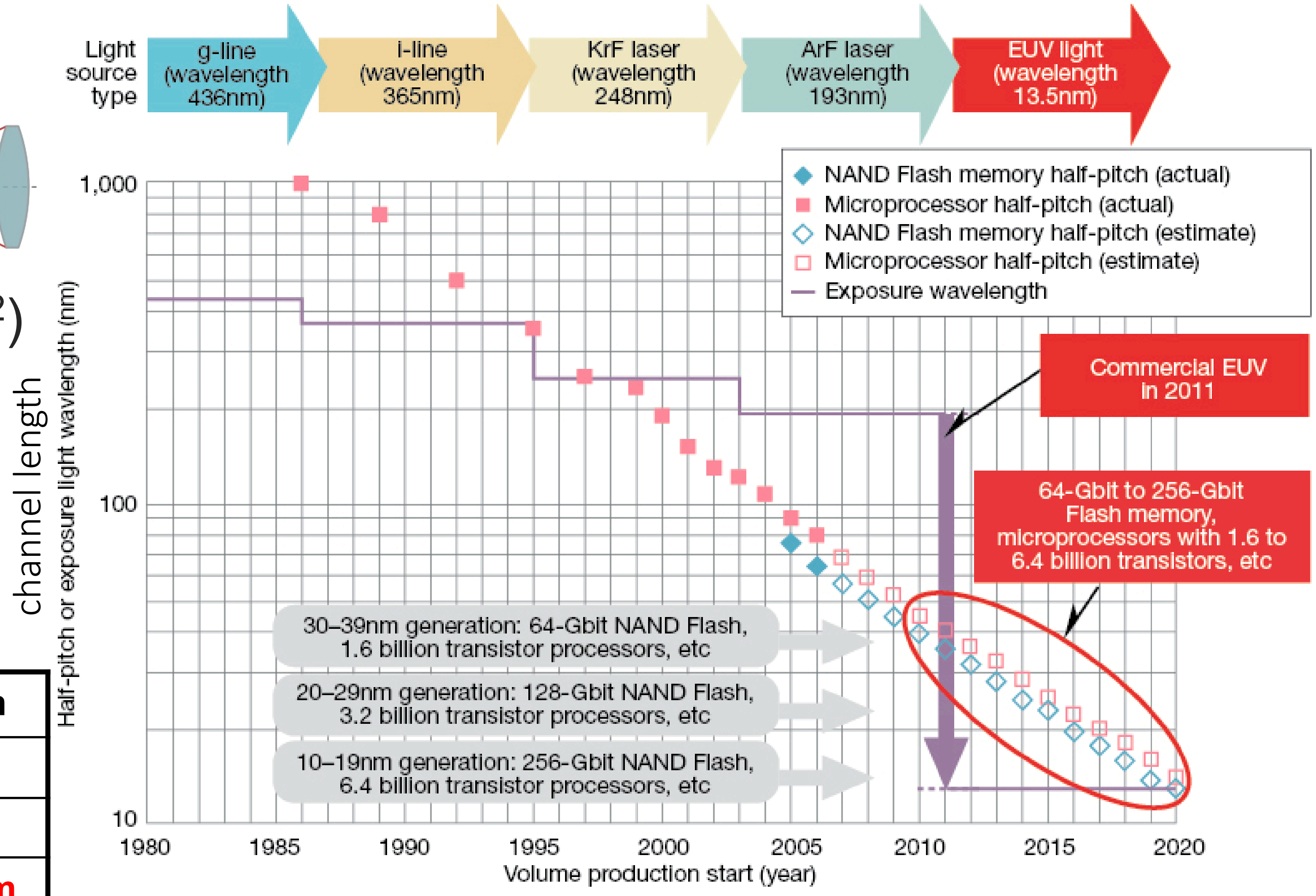
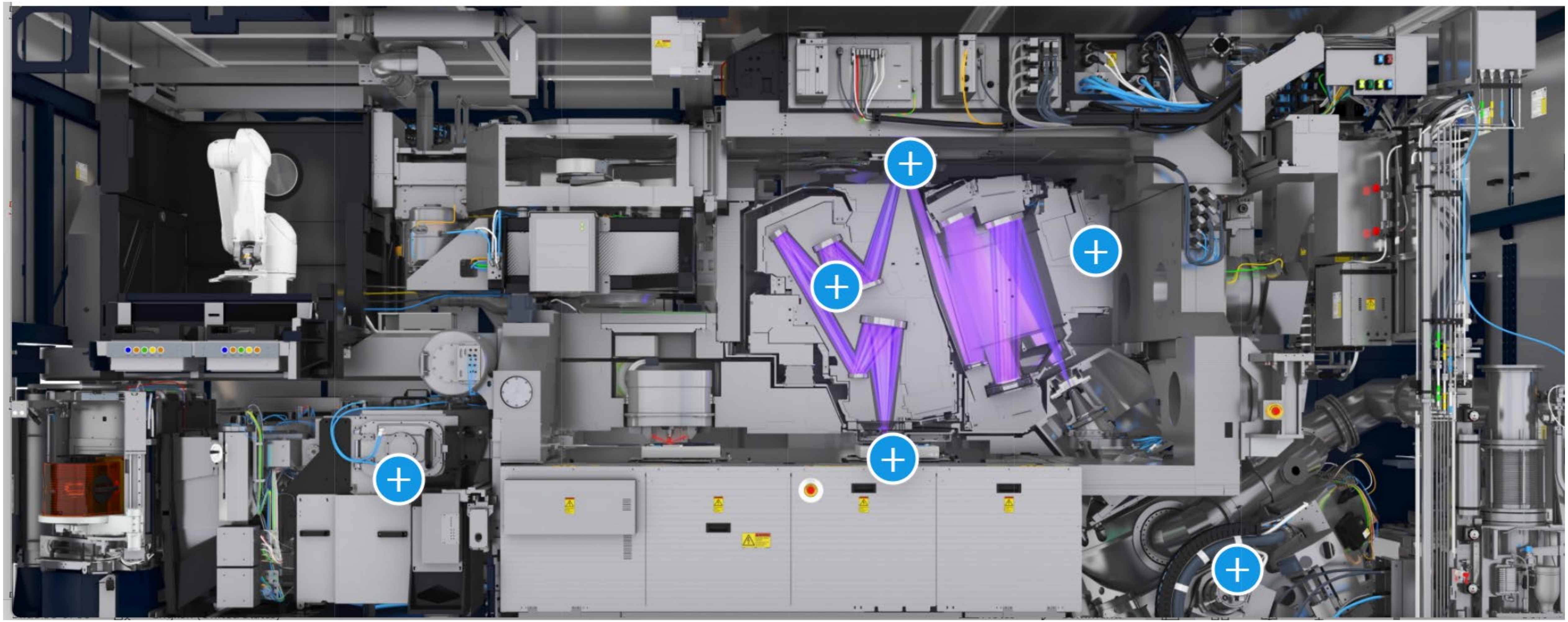


Diagram by Nikkei Electronics based on materials from Intel, International Technology Roadmap for Semiconductors (ITRS), etc. http://www.newmaker.com/news_41958.html

Nanofabrication

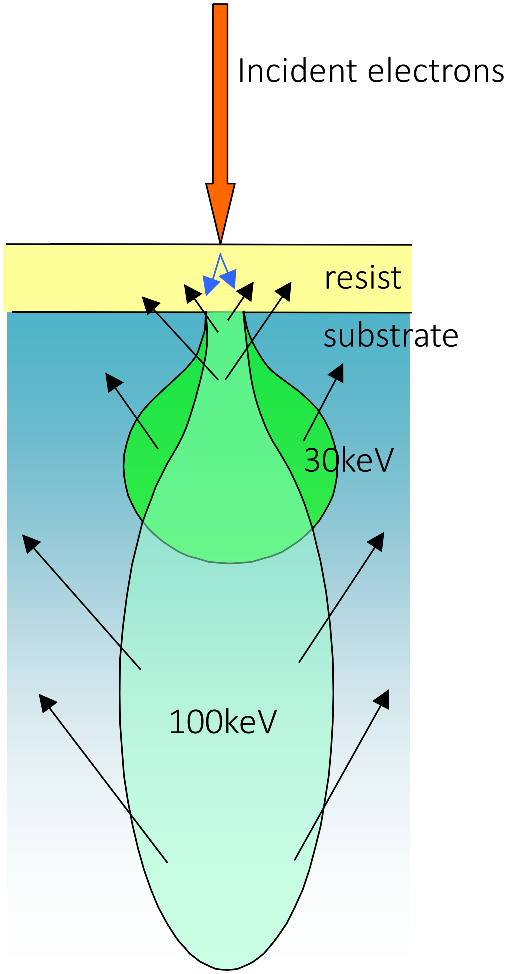
ASML EUV system NXE:3400B (ref: ASML website)

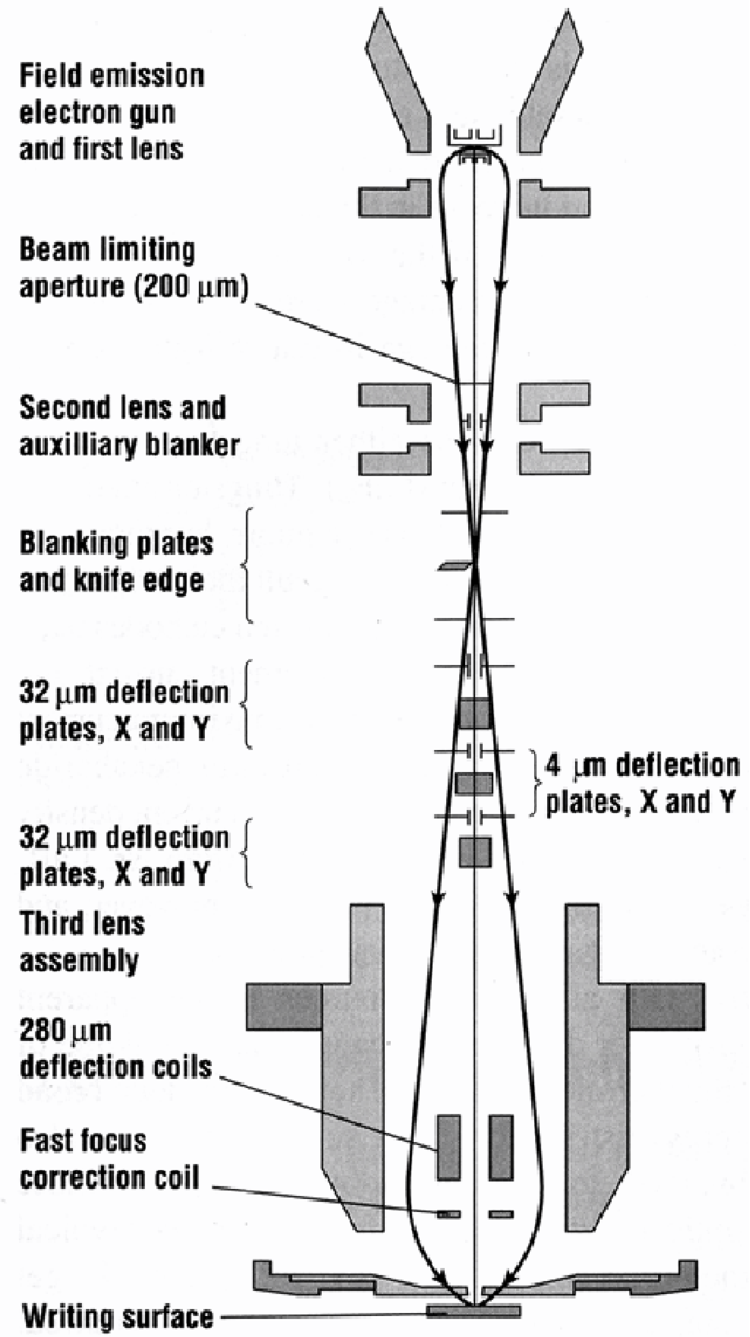


Nanofabrication

e-beam lithography

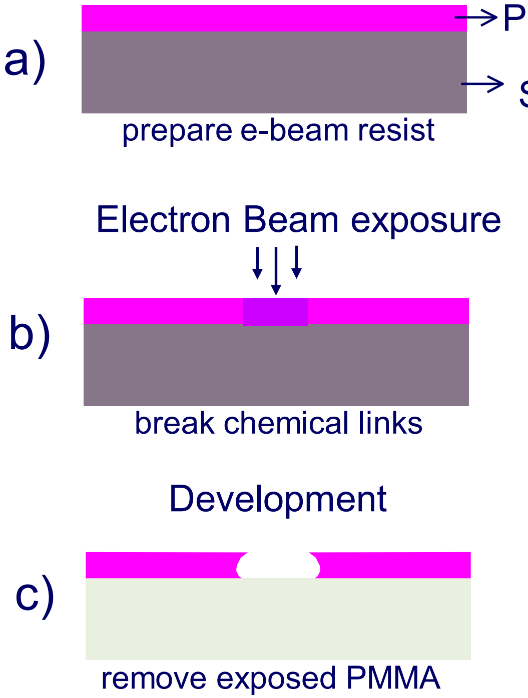
- No mask is needed! (both pros and cons)
- the wavelength of electrons can be controlled, therefore fine structures can be realized.
- However, compared to photolithography, it is a much slower process.
- Incident electron energy, current can control the spot size of the beam, therefore, influence the feature as well as speed.
- It is very useful in research but not industry.

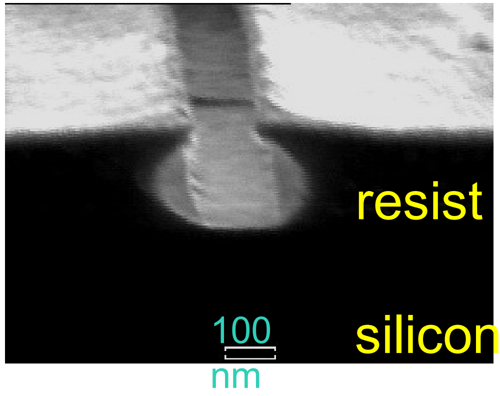


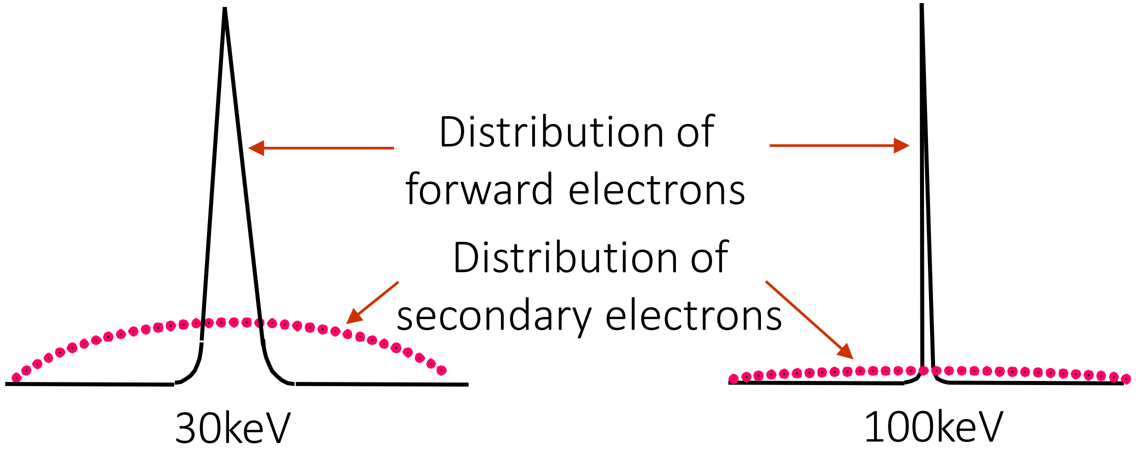


Nanofabrication

e-beam lithography







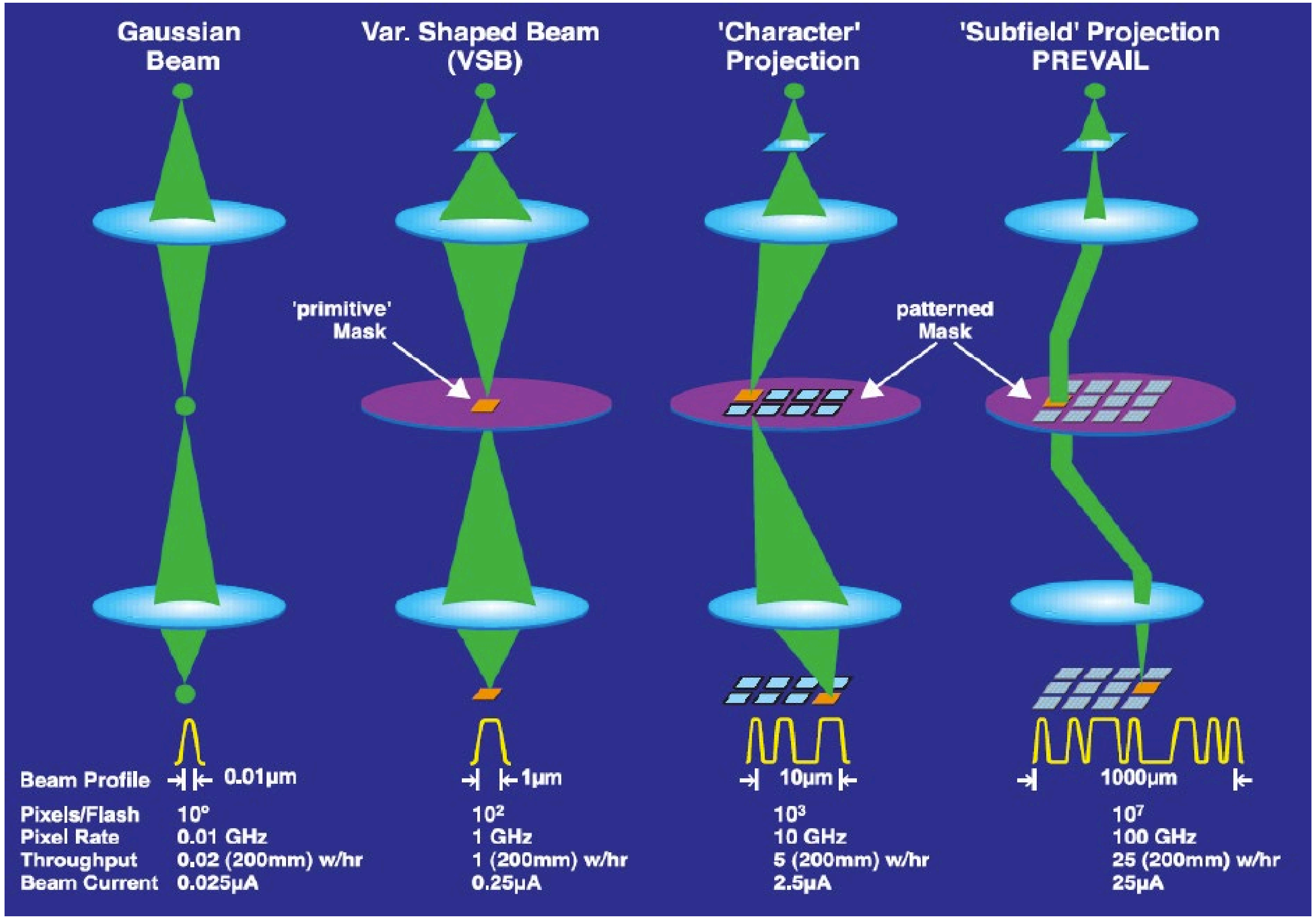
30keV:

1. Good for the prototype test
2. Thin resist line-width < 30nm
3. Clear align key image
4. Good for lift-off process
5. Lack of stage stability

100keV:

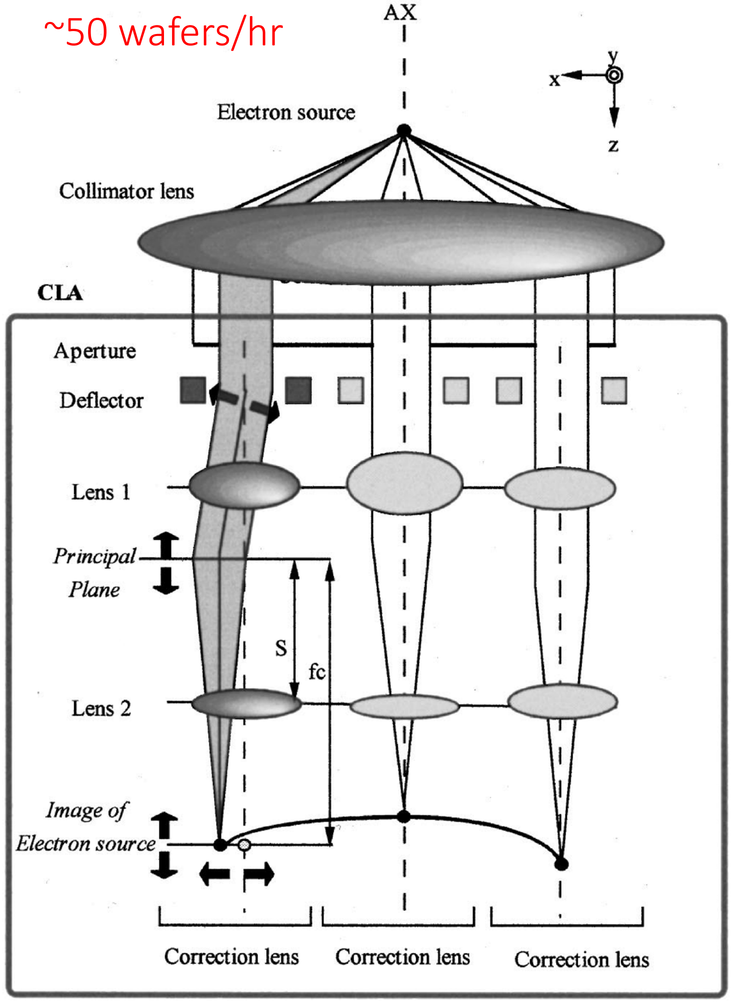
1. Good for large-area exposure
2. Thin resist line-width < 10nm
3. Require thick/clear align keys
4. Require extra resist engineering
5. Stable/accurate stage stability

Nanofabrication



楊富量 (NDL), Outlook for 15nm CMOS Manufacture

~50 wafers/hr



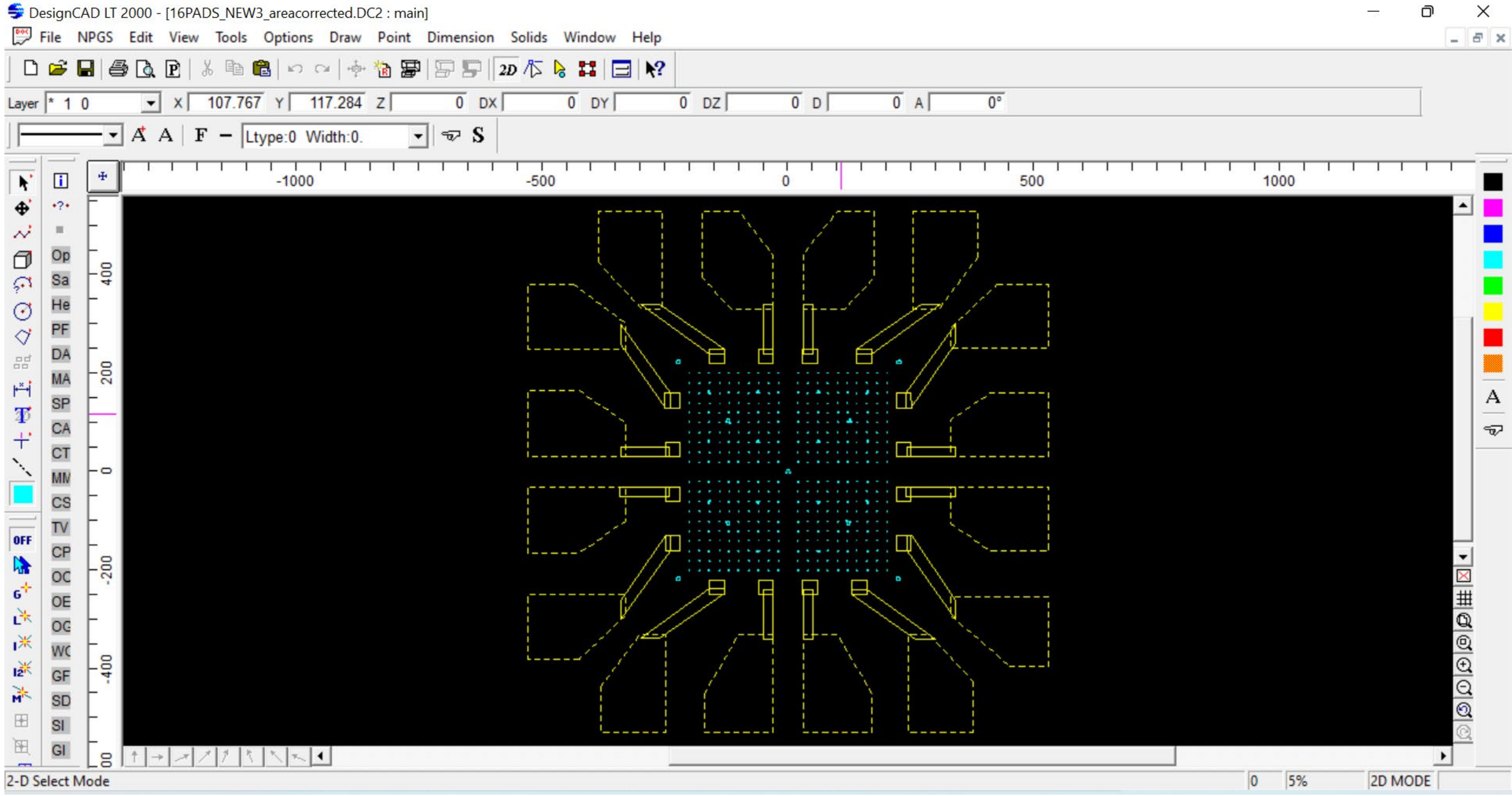
M. Muraki et al. J. Vac. Sci. Technol. B 18(6), 3061, 2000, *Canon Inc.*,

Nanofabrication

E-beam lithography

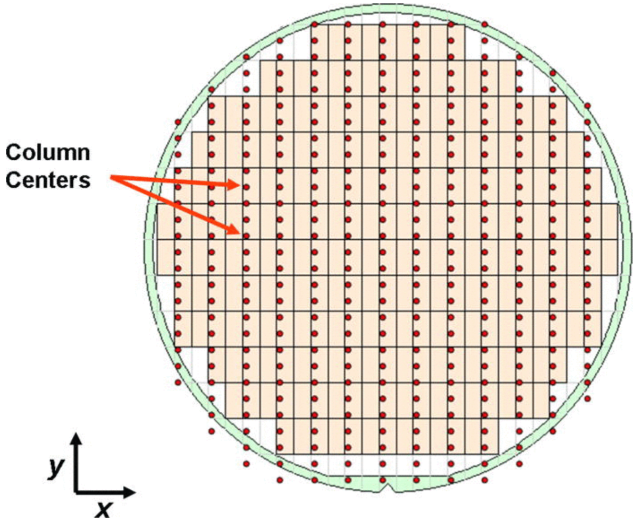
One also needs to generate the patterns for which you would like the e-beam system to write.

There is a lot of software for this purpose such as AutoCAD, K-layout, DesignCAD, etc...



Nanofabrication

Beam voltage: 50- 150 kV
Beam current: 10th pA to 800 nA
Field size: 0.5x 0.5 to 300x300 mm^2



Dan Meisburger et al.
doi.org/10.1116/1.4931589

E-beam lithography

RAITH

Elionix

Nanofabrication

Characterizations:

AFM,

profilometer,

electrical measurement,

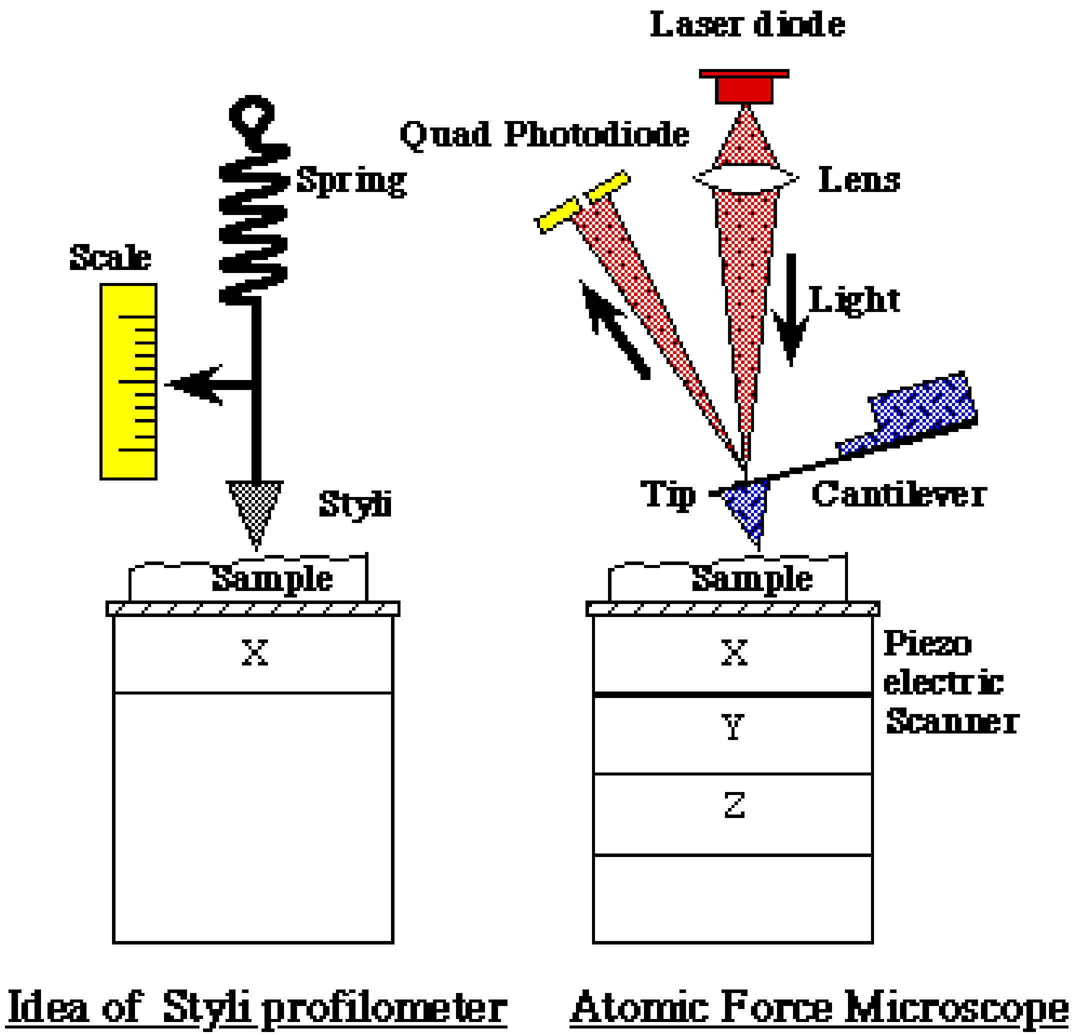
X-ray,

Photoluminescence,

Raman spectrum,

ARPES,

SEM/TEM



https://probe.olympus-global.com/en/product/spm/

Nanofabrication

Characterizations:

AFM,

profilometer,

electrical measurement,

X-ray,

Photoluminescence,

Raman spectrum,

ARPES,

SEM/TEM

Source: Bruker, Park

Nanofabrication

Characterizations:

AFM,

profilometer,

electrical measurement,

X-ray,

Photoluminescence,

Raman spectrum,

ARPES,

SEM/TEM

https://ayscomdatatec.com/en/probe-stations/

https://www.photonicsonline.com/

One can measure C-V, I-V, thermal properties, etc.

Nanofabrication

Characterizations:

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X-ray,

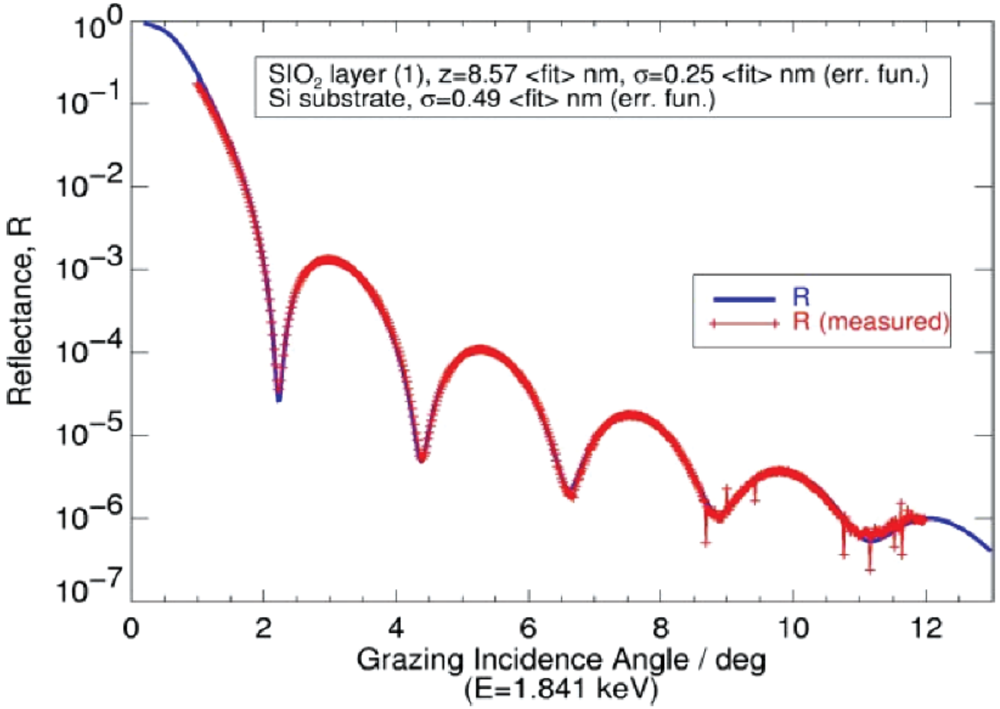
Photoluminescence,

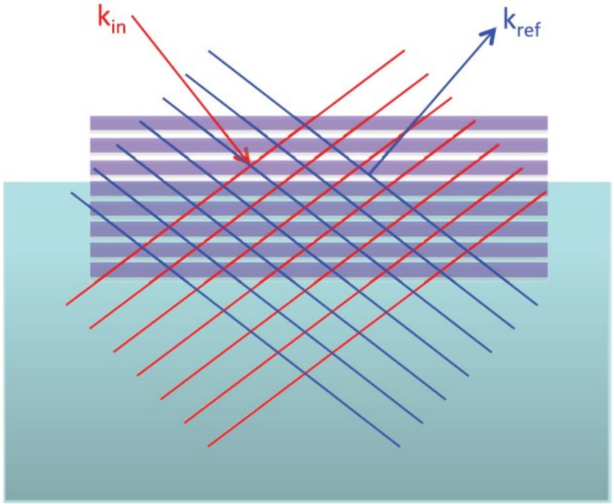
Raman spectrum,

ARPES,

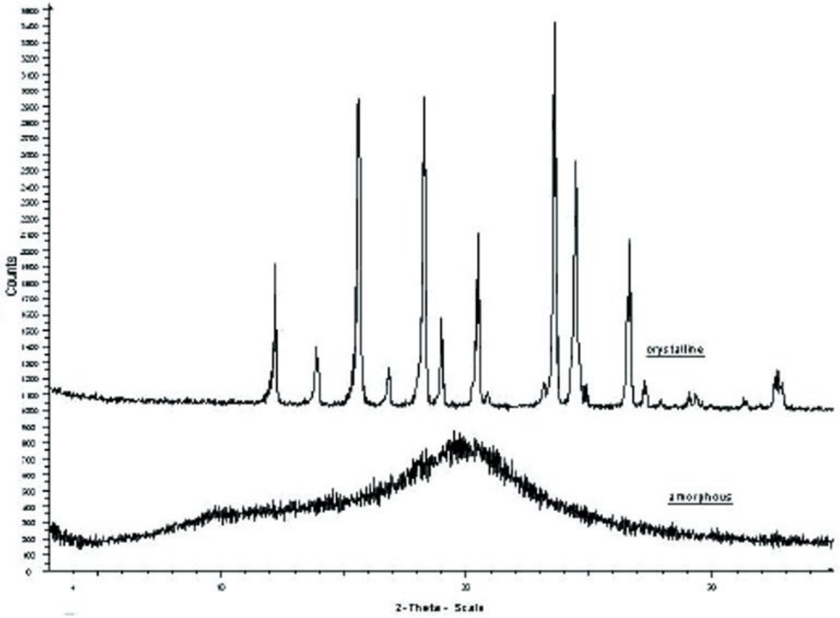
SEM/TEM

Reflection





Nanofabrication



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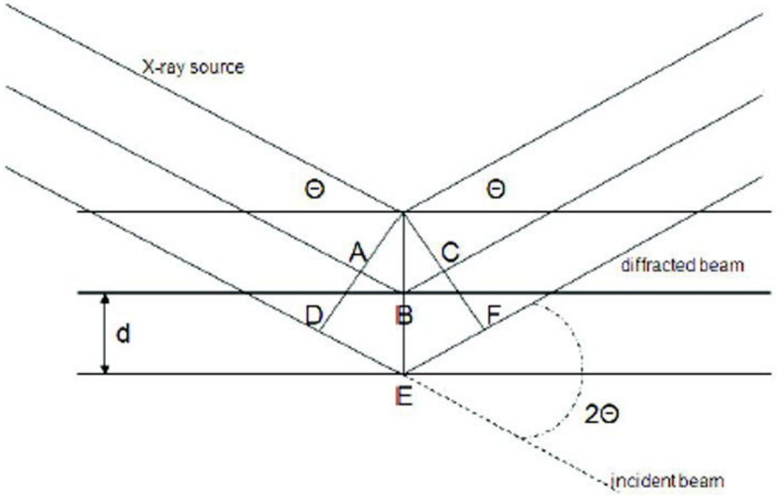
Photoluminescence,

Raman spectrum,

ARPES,

SEM/TEM

Powder diffraction



Source: IOP and Anthony L. Young

Nanofabrication

Characterizations:

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X-ray,

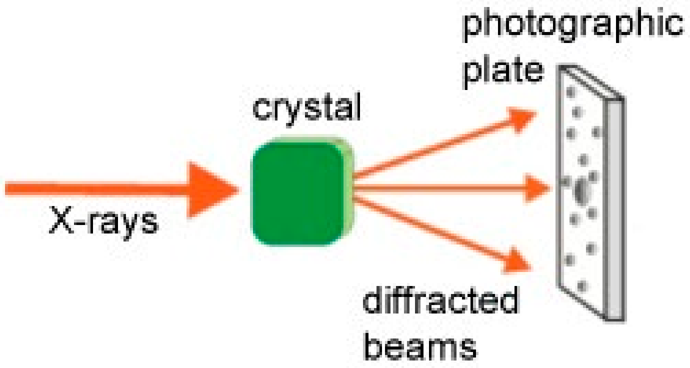
Photoluminescence,

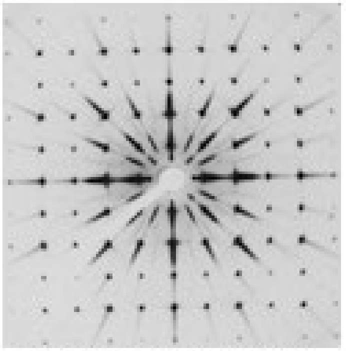
Raman spectrum,

ARPES,

SEM/TEM

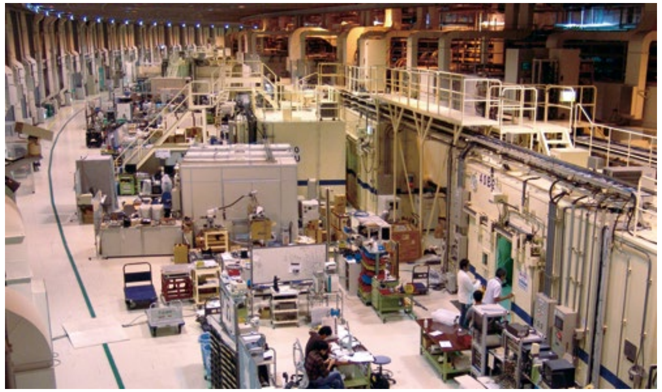
Transmission

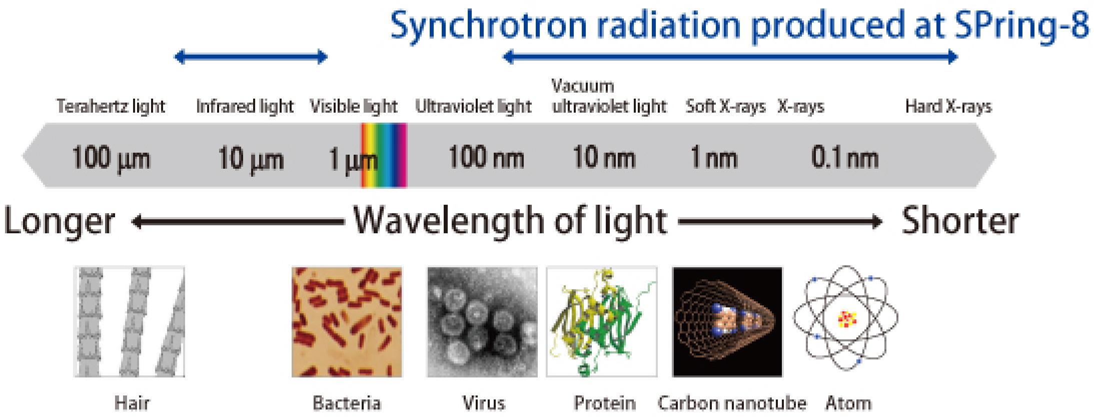




difraction pattern

Spring 8





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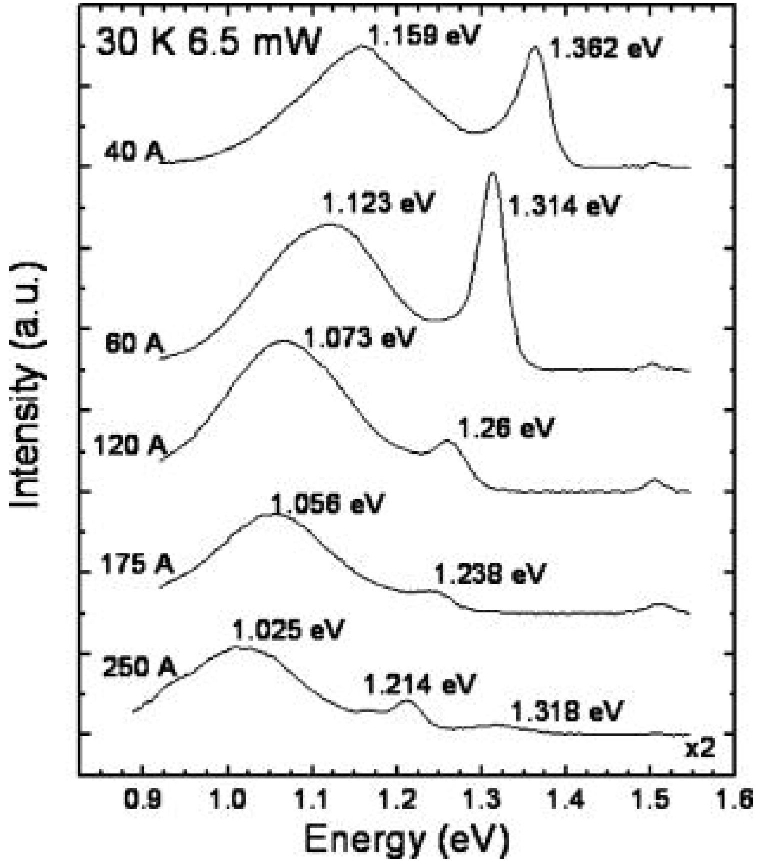
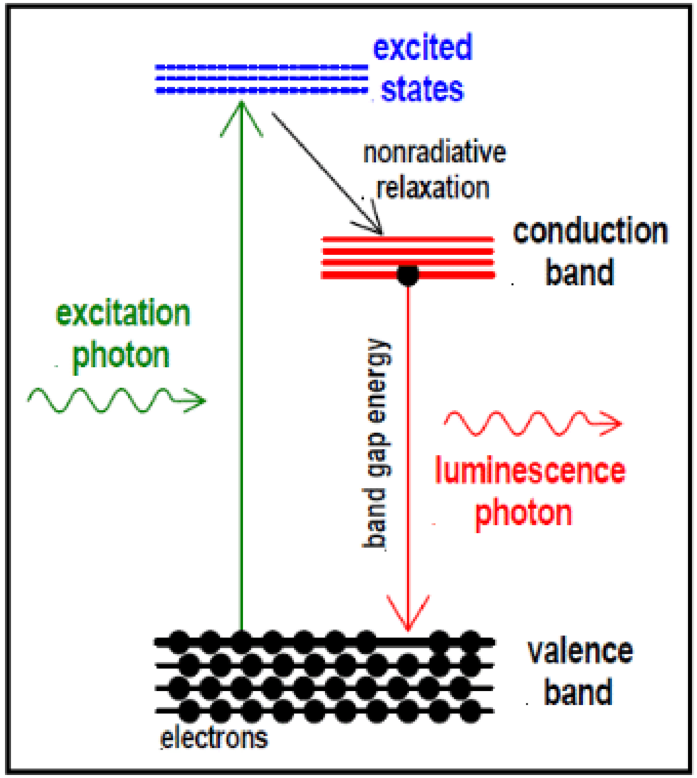


JF Chen, et al, JAP 2007



Phuong Vuong, thesis

Raman, an inelastic process in which photon electron transfer to molecule

Nanofabrication

Characterizations:

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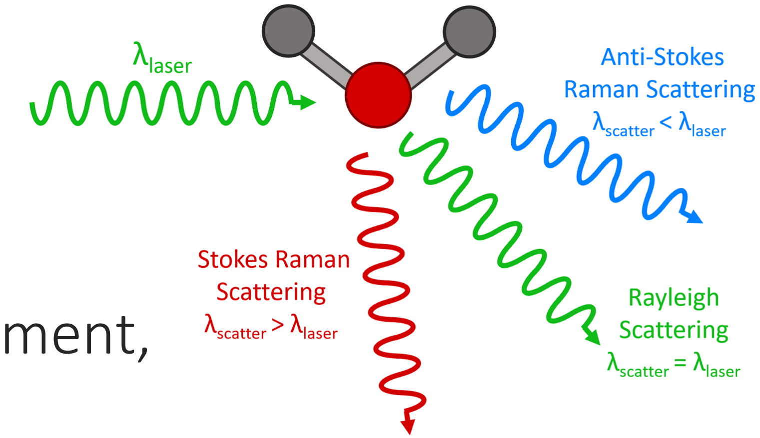
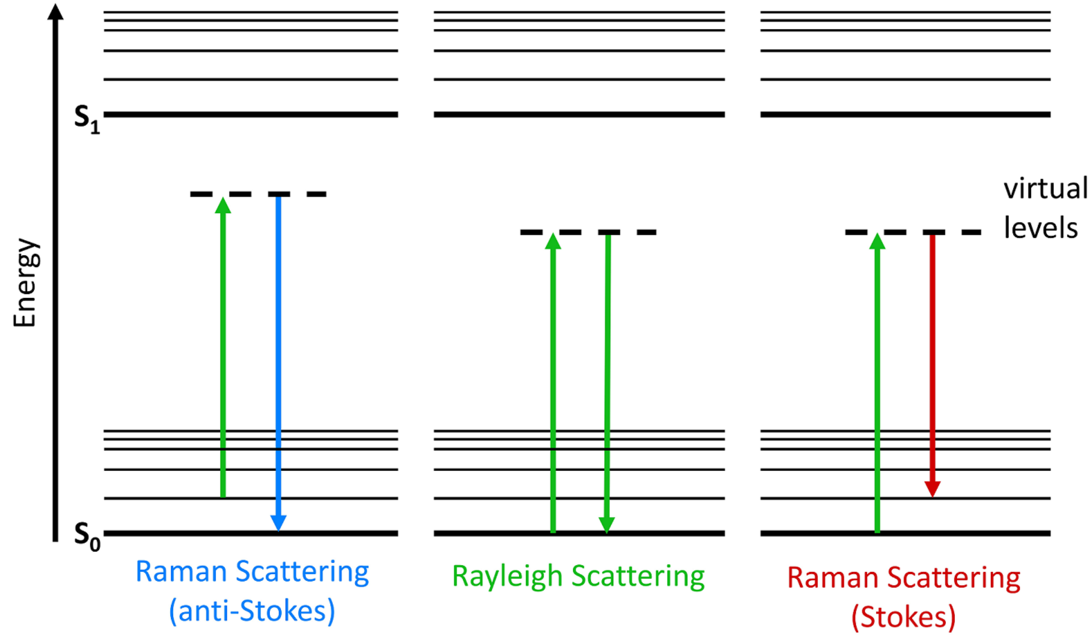
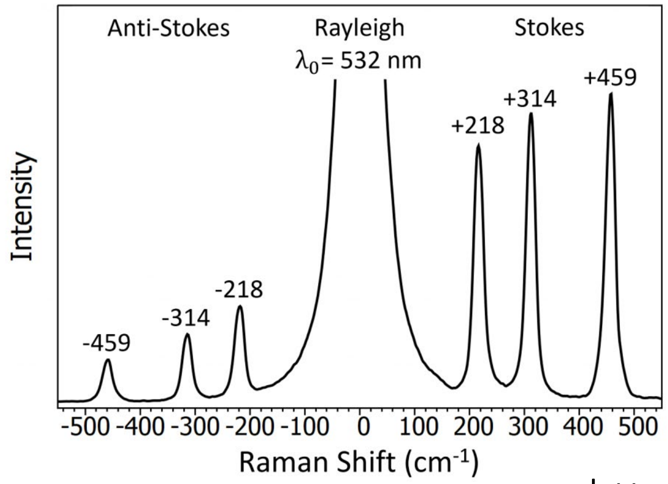
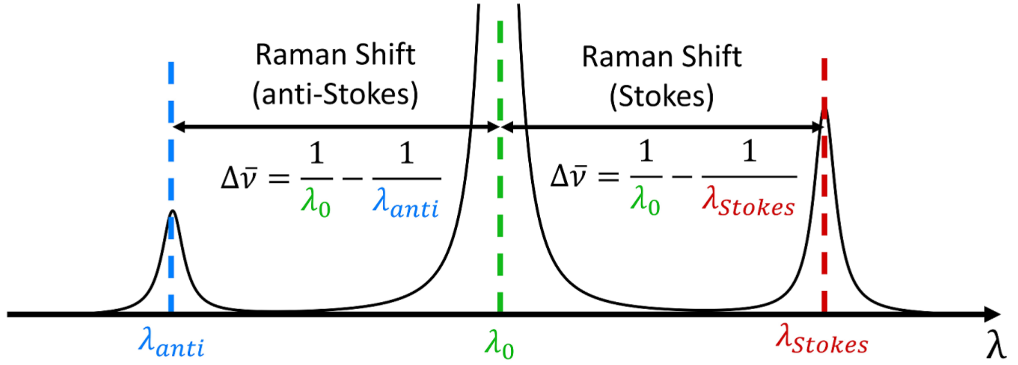
SEM/TEM



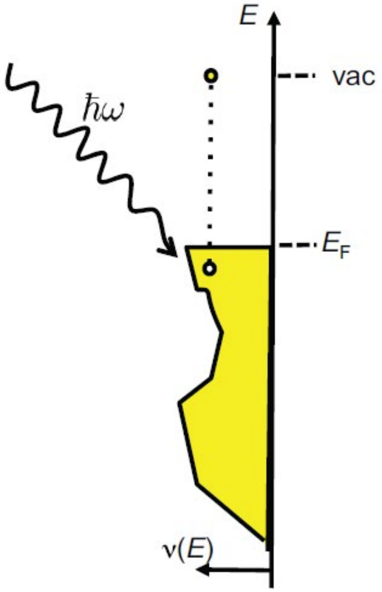


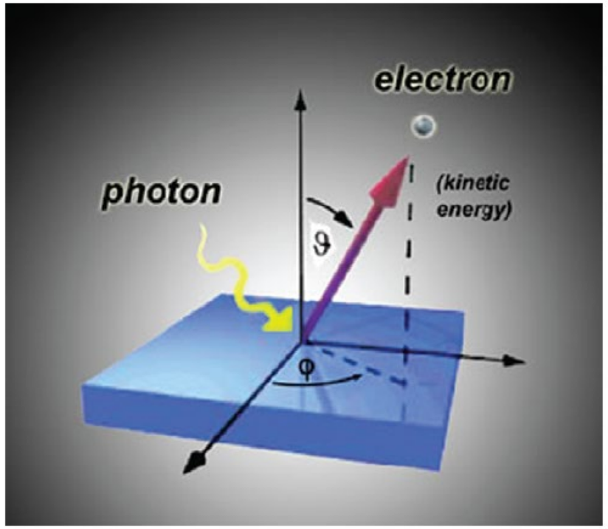




https://www.edinst.com/blog/what-is-raman-spectroscopy/

Nanofabrication





Characterizations:

AFM,

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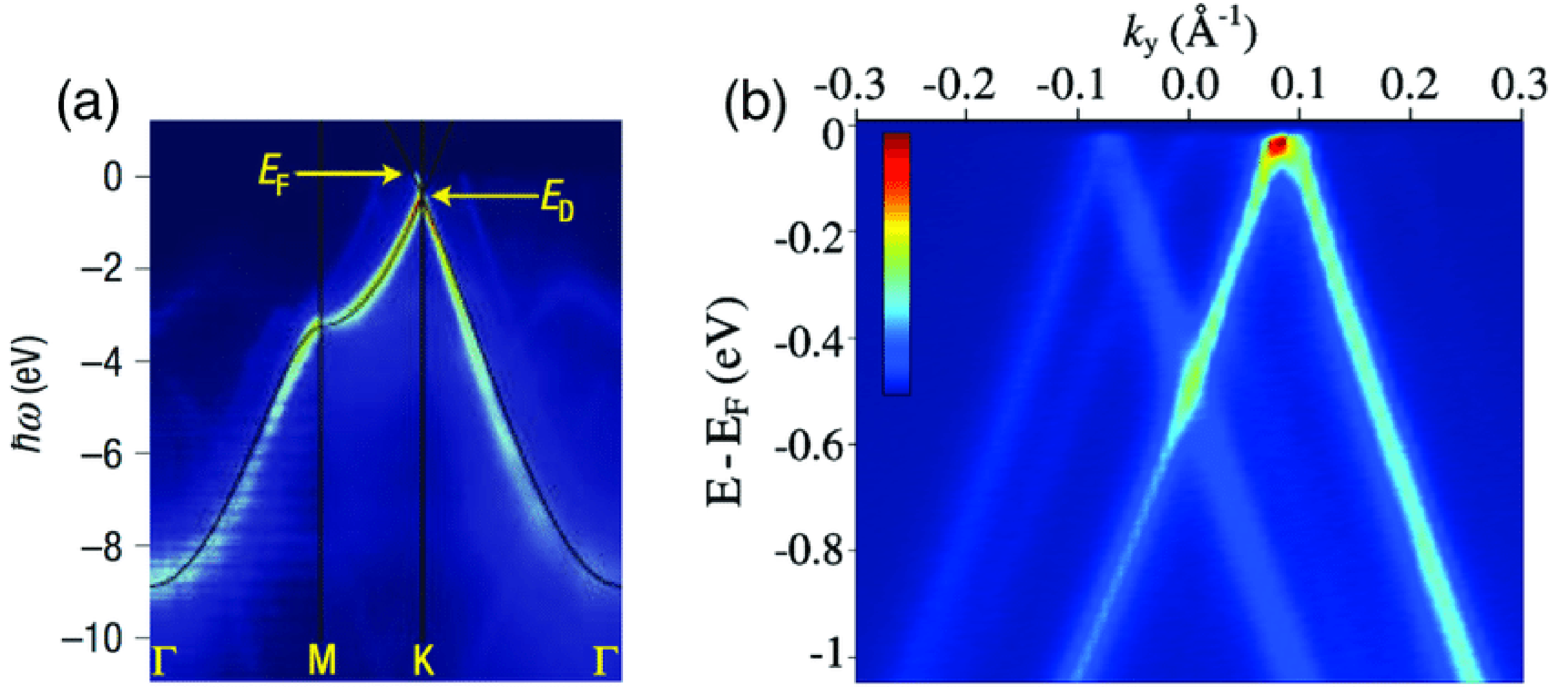
X-ray,

Photoluminescence,

Raman spectrum,

ARPES,

SEM/TEM



T. O. Wehling et al, DOI:10.1080/00018732.2014.927109 (2014)

Nanofabrication

Characterizations:

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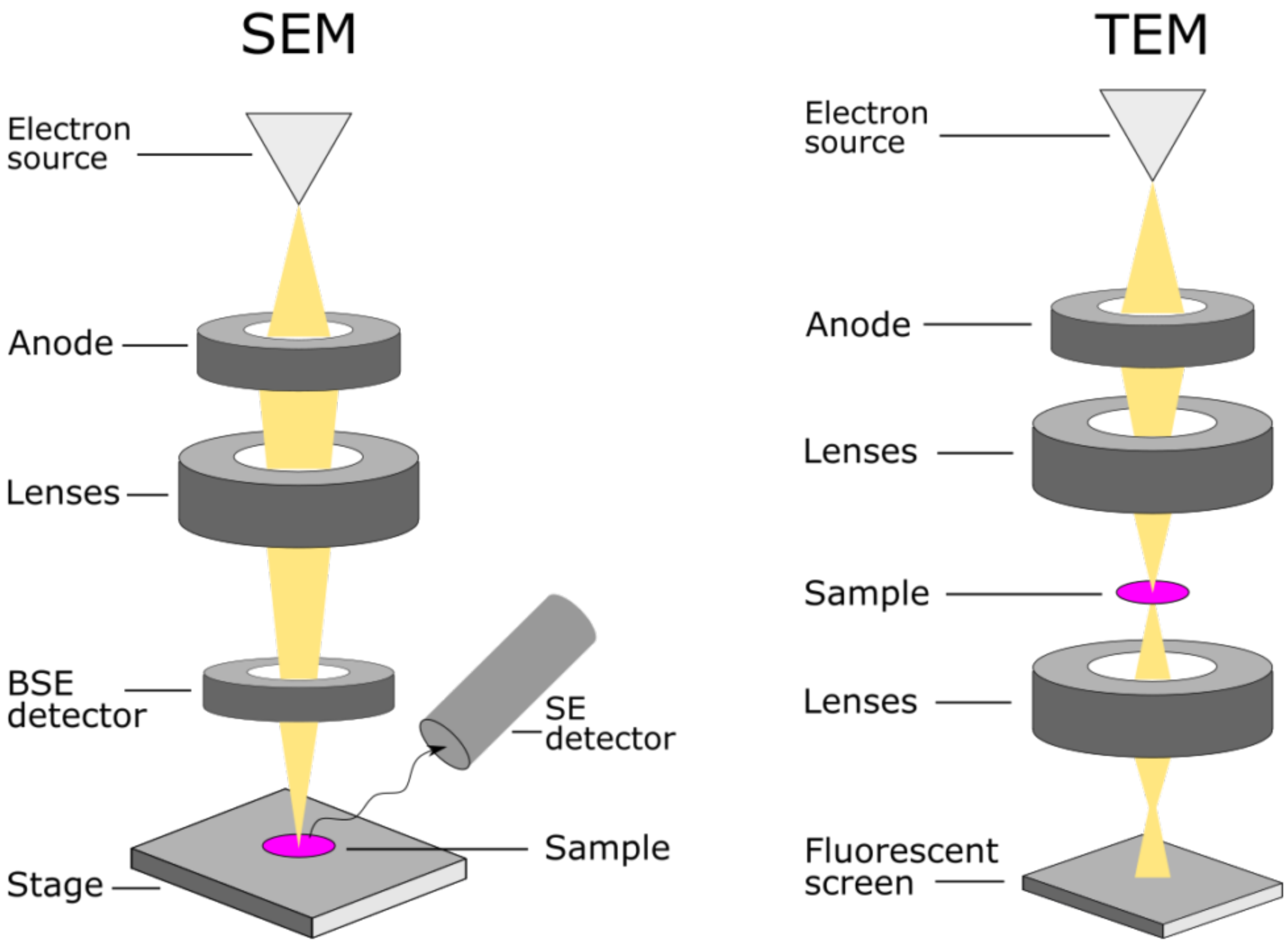

https://anapath.ch/electron-microscopy-2/

Nanofabrication

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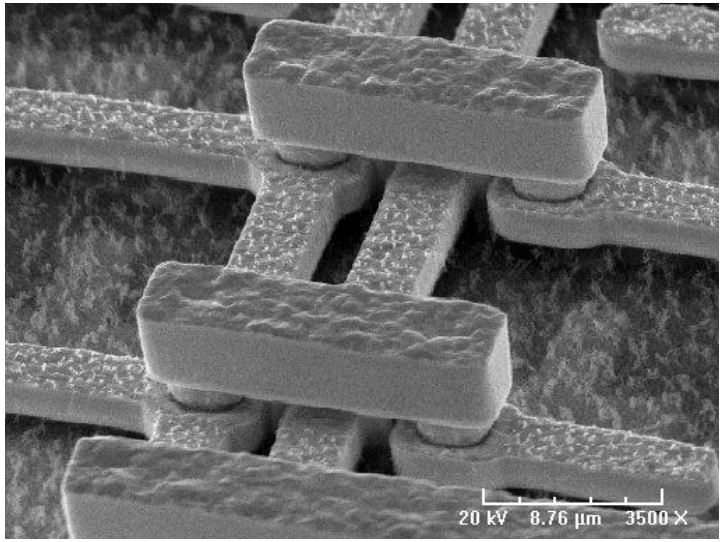
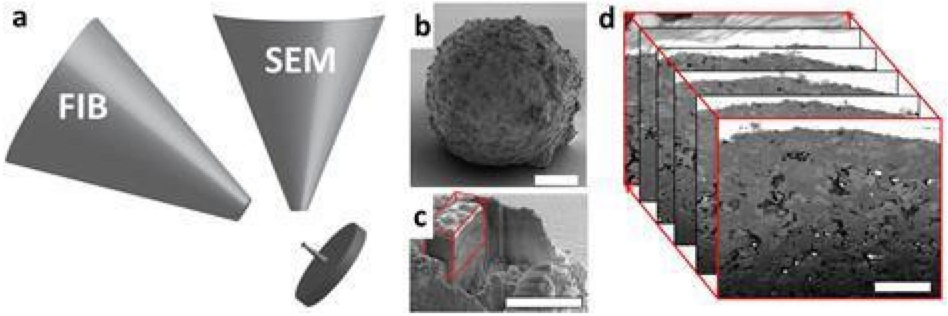
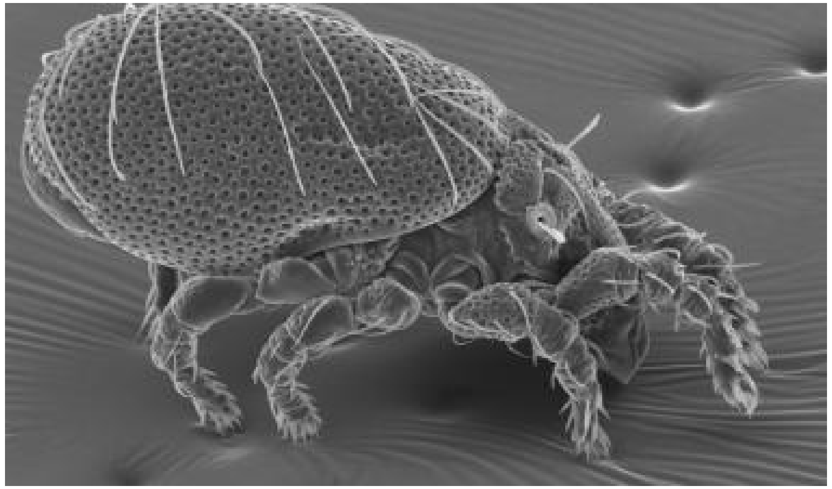


https://orslabs.com/services/material-analysis/sem/

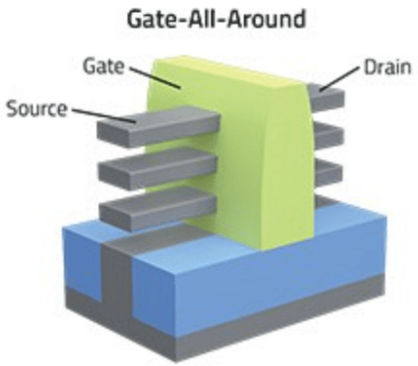


Bert Weckhuysen / Utrecht University



https://www.huttonltd.com/services/scanning-electron-microscopy-sem

Nanofabrication



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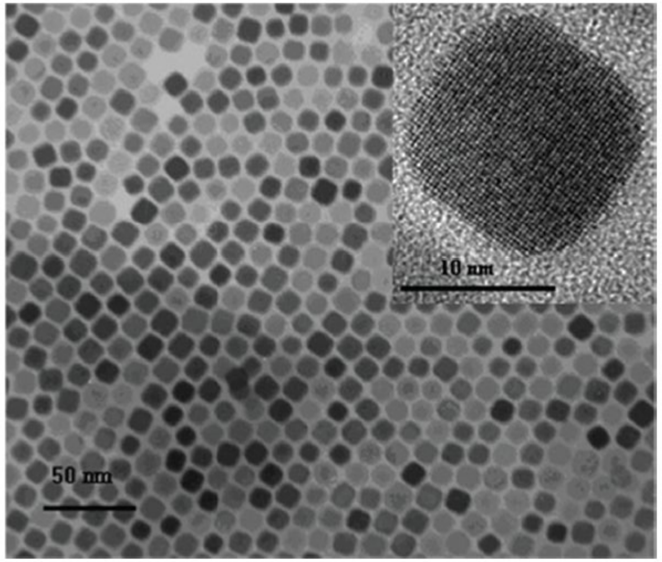
Photoluminescence,

Raman spectrum,

ARPES,

SEM/TEM/STM



PbS nanocrystals as imaged via TEM

IBM STEM EDX image